Volatility and Growth: the Case of the U.S. and U.K.*

Jang-Ryoul Kim, Gie-Young Lim

차 례

I. Introduction and Motivation

The linkage between the growth rate of an economy and the volatility of that growth rate has long been a subject of intense scrutiny. In contrast to the sheer volume of previous studies on this subject, however, there has been no clear theoretical consensus on this topic. One view is that the relationship should be positive. Schumpeter suggests that fluctuations in economic activity improve the efficiency of the economic system, thus improving the long-term growth via "constructive destruction". Black (1987) focuses on the positive correlation between volatility and return, arguing that agents choose to invest in riskier technologies only if the

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expected rates of return (i.e., growth rate) are high enough to compensate for the associated higher risk.

Another view, arguing for a negative relationship, is found in Woodford (1990), Ramey and Ramey (1991), and Martin and Rogers (2000). Ramey and Ramey (1991) argue that if firms have to decide on the technological methods used in advance, then the higher uncertainty due to higher volatility renders output lower. In a similar vein, Martin and Rogers (2000) hypothesize that a negative relationship exists when learning-by-doing is the main driving force for the long-run growth.

The lack of theoretical consensus renders the question inevitably an empirical one. However, statistical evidence on the linkage between volatility and growth is also ambiguous. To name a few cross-sectional studies, Grier and Tullock (1989) find a positive relation while Ramey and Ramey (1995) and Martin and Rogers (2000) report a negative relation. Among time-series studies, Caporale and Mckiernan (1996, 1998) find a positive relation for UK and US, whereas Henry and Olekalns (2002) find a negative relation for Australia and US. Several other studies, including Speight (1999) and Grier and Perry (2000), discover no significant relation for UK and US.

What is worth noting is that most, if not all, of previous empirical studies postulate that the relation between volatility and growth is stable across the states of the economy.

More specifically, most empirical models implicitly assume that the sign and size of the volatility-growth relation are the same when the economy is in recession (or low-growth state) or expansion (or high-growth state). There is no a priori reason, however, to believe that is the case. Rather, it is conceivable that the sign of the volatility-growth relation depends on whether the economy is on recession or expansion. One plausible scenario would go as follows: suppose that two mechanisms for technological growth exist in the economy. One is the internal (or purposeful) learning that substitutes for production activities in the spirit of Aghion and Saint-Paul (1998 a, b). The other is the external (or serendipitous) learning as in Martin and Rogers (1997, 2000), which is complementary to production activities. Also suppose that the former (or latter) form of learning process works mainly when the economy is in expansion (or contraction). Under these circumstances, the average productivity and the rate of growth in expansion increase as volatility increases, while in recession the increased volatility decreases growth rate.[1)]

Based on this insight, we seek to explain the lack of robust empirical evidence on the volatility-growth relation from a different perspective. Our point of departure is the conjecture that the sign of the relation is positive when the economy is in expansion and negative in recession. We set off with the conjecture that higher volatility exerts adverse effects on growth when the economy is in contraction and favorable effects in expansion. If the conjecture is right, then empirical investigations of data without taking this feature into account will fail to recover non-trivial relation between volatility and growth. We estimate a series of ARCH-type models with the real GDP data of the U.S. and the U.K., and find strong evidence suggesting that the volatility-growth relation is positive when the economy is in expansion, while higher volatility lowers economic growth rate in the contraction phase.[2)]

Our results have some implications for stabilization policy. During the recessions when the relation is likely to be negative, a policy designed to

1) See Caballero and Hammour (1994) and Blackburn (1999) for more detailed argument on the types of learning and their effects on growth.

2) There are a few pieces of empirical evidence on the asymmetric relation between volatility and growth across the state of the economy. Using the threshold GARCH (TGARCH) model for the U.S. industrial production, McMillan and Speight (1998) find greater impact of negative shock on output volatility.

decrease business cycle fluctuations are also consistent with the goal of high long−run growth. When the economy is expanding, however, then any policy to stabilize the business cycle may come at the cost of slower long−run economic growth.

The paper is organized as follows. In section II, the results of pretests for the asymmetric effects of volatility on growth are presented. Section III presents the Markov−switching model in which the sign of volatility−growth relation is regime dependent. In section IV, the estimation results are reported for the model developed in the previous section. Section V concludes the paper.

II. Volatility vs. Growth : Pretests with ARCH-type Models

In this section, we run a series of pretests for the presence and nature of the relation between volatility and growth in the US quarterly GDP.[3] We opt to fit ARCH−type models to data, because such models generally provide consistent estimates of the time−varying residual conditional variances of the series in question. Based on the estimation results, we figure out whether there are enough variations in the volatility of the output growth, and whether the sign of the volatility−growth relation is invariant orchanges over business cycle. We set off by estimating the following autoregressive models for US quarterly GDP growth

$$Y_t = \Phi_0 + \Phi_1 Y_{t-1} + \dots + \Phi_p Y_{t-p} + \xi_t \qquad (1)$$

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3) We use the seasonally adjusted real GDO series of the U.S. over 1960:Q1−2009:Q3, available from the Federal Reserve Bank of St. Louis database.

and apply the ARCH−effect test of Engle (1983) to the estimated residuals $\hat{\xi}_t$.The results are given in Table 1 below, where the ARCH effects are clearly detected for p = 2 to 5 at the 5% significance level.

Table 1: Test Results for ARCH effects

	p=2	p=3	p=4	p=5
LM test	7.031(0.030)	6.797(0.033)	6.729(0.035)	7.419(0.024)

Note: The LM test statistic follows $x^2(p)$ distribution under the null hypothesis of no ARCH effects. Numbers in parentheses are p−values.

Having confirmed the presence of ARCH effect in the residuals of equation (1), we proceed to construct the conditional variances of { ξ_t } and check if they show variations large enough to nontrivially affect output growth. To do so, we formally estimate the following GARCH(1,1) model

$$Y_t = \Phi_0 + \Phi_1 Y_{t-1} + \dots + \Phi_p Y_{t-p} + \xi_t \qquad (2a)$$

$$\xi_t \mid \Omega_{t-1} \sim N(0, h_t), \quad h_t = \alpha_0 + \alpha_1 \xi_{t-1}^{\ 2} + \alpha_2 h_{t-1} \qquad (2b)$$

where Ω_{t-1} denotes the information set up to the period t−1 and suitable restrictions are imposed on the coefficients in the variance equation (2a).[4] The estimated versions of equations (2a)−(2b) are given below:

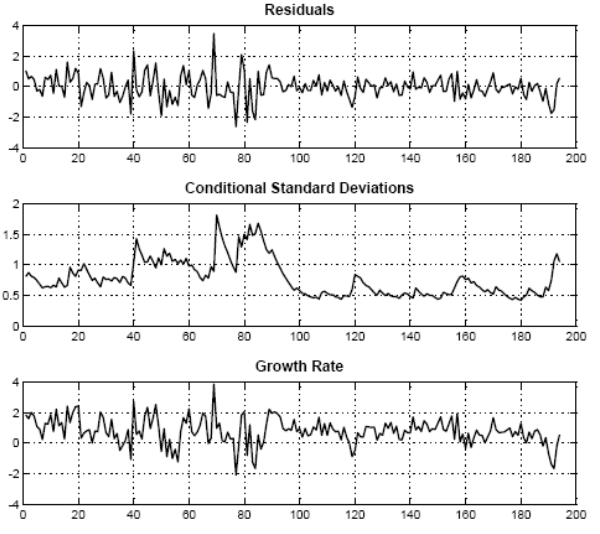
$$\underset{}{Y_t} = \underset{(4.311)}{0.435} + \underset{(3.134)}{0.265\,Y_{t-1}} + \underset{(3.398)}{0.296\,Y_{t-2}} - \underset{(-0.936)}{0.069\,Y_{t-3}} + \xi_t \qquad (2a')$$

4) The order 3 of autoregression is determined by the Ljung−box statistic from the equation (1). Results for other values of p, which are qualitatively similar, are available from the authors upon request.

$$h_t \;=\; \underset{(1.376)}{0.025} \;+\; \underset{(3.029)}{0.221\xi_{t-1}^{\,2}} + \underset{(13.060)}{0.765h_{t-1}} \qquad \text{(2b')}$$

where t−values of the estimates are given in the parentheses. The estimation results above clearly show the existence of strong autoregressive and GARCH effects in the mean and variance equation, respectively. More specifically, the ARCH and GARCH coefficients are both sharply estimated at 5% critical level, implying considerable degree of volatility clustering and inertia in conditional variances. This finding is illustrated in [Figure 1] below, which plots the estimates of residuals $\{\xi_t\}$ and conditional standard deviations $\{h_t^{\ 1/2}\}$, along with the growth rate series.

[Figure 1]: Plots of Disturbances and Conditional Volatilities



With the estimated volatility series in hand, the next step is to check if the output volatility affects output growth significantly. For this aim, we estimated the following model

$$Y_t = \Phi_0 + \Phi_1 Y_{t-1} + \ldots + \Phi_p Y_{t-p} + \gamma h_t^{1/2} + \xi_t \tag{3a}$$

$$\xi_t \mid \Omega_{t-1} \sim N(0, h_t),\ h_t = \alpha_0 + \alpha_1 \xi_{t-1}{}^2 + \alpha_2 h_{t-1} \tag{3b}$$

where the mean equation (3a) now includes the conditional standard deviation $\{h_t^{1/2}\}$ as another explanatory variable. The estimation results for the GARCH-M model above are given below, with t-values in parentheses:

$$\underset{}{Y_t} = \underset{(4.311)}{0.402} + \underset{(3.134)}{0.265} Y_{t-1} + \underset{(3.398)}{0.296} Y_{t-2} - \underset{(-0.936)}{0.069} Y_{t-3} + \underset{(0.14)}{0.026} h_t^{1/2} + \xi_t \tag{3a'}$$

$$h_t = \underset{(1.354)}{0.025} + \underset{(2.999)}{0.222} \xi_{t-1}^2 + \underset{(12.960)}{0.764} h_{t-1} \tag{3b'}$$

Results in equations (3a')-(3b') are virtually identical to those in equations (2a')-(2b'), and show that the conditional volatility of output has no effect on output growth, statistically or economically. Therefore, the results for the GARCH-M model seemingly fail to provide a strong support of a non-trivial relation between volatility and growth. As mentioned in the introduction, however, it is conceivable that this failure is attributable to the misspecification of the empirical model, ignoring the changes in the nature of the relation across the states of the economy. If the sign of volatility-growth relation switches across the expansion and contraction phases of the business cycle, then attempts to construct the relation without taking the switches into account are likely to fail in recovering the significant effects of volatility on output.

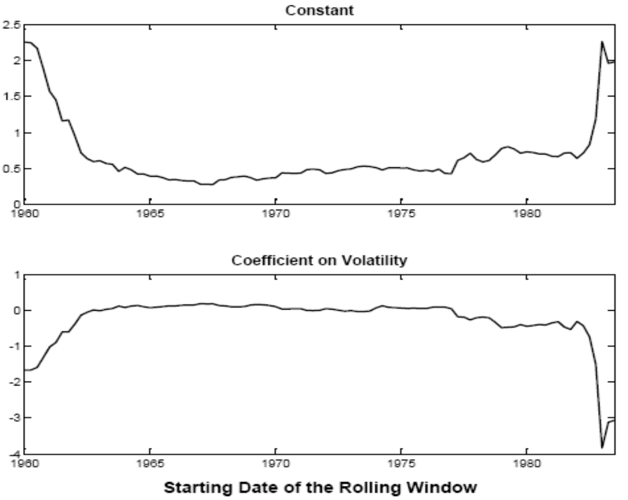
III. Business Cycle Dependence of the Volatility-Growth Relation

1. Preliminary Check on the Conjecture

Before formally addressing the business cycle dependence of the volatility−growth relation, we perform a preliminary check on its validity. We consider a rolling regression, in which samples in consecutive twenty−five year rolling windows are used to estimate the equations (3a)−(3b). Here, we expect that the plots of the estimated constant (which corresponds to the average growth rate over the sample in each window) and the coefficient on the volatility term will vary in opposite directions, provided that our conjecture is valid.

[Figure 2] below plots the series of the two estimates form the rolling regression. Strikingly enough, the movements of the two estimatesare almost the mirror images of each other, with the simple correlation coefficient between the two series amounting to −0.911. In summary, the results of rolling regressions support the business cycle dependence of the volatility−growth relation. We will develop this idea further in the next subsection.

[Figure 2]: Constant and the Coefficient on Volatility from the Rolling Regression



2. A GARCH-M Model with Business Cycle Dummy

We now search for more rigorous empirical evidence on the business cycle dependence of the volatility-growth relation. To do so, we divide the whole sample into the high-growth periods and low-growth periods, and construct dummy variable $\{D_t\}$ for the high-growth periods. Then we estimate the following model:

$$Y_t = \Phi_0 + \Phi_1 Y_{t-1} + \ldots + \Phi_p Y_{t-p} + \mu_1 D_t + (\lambda_0 + \lambda_1 D_t) h_t^{1/2} + \xi_t \tag{4a}$$

$$\xi_t \mid \Omega_{t-1} \sim N(0, h_t), \quad h_t = \alpha_0 + \alpha_1 \xi_{t-1}{}^2 + \alpha_2 h_{t-1} \tag{4b}$$

where the additional explanatory variables D_t and $D_t \cdot h_t{}^{1/2}$ in the mean equation (4a) are intended to capture the business cycle dependence of the average growth rate and the volatility-growth relation, respectively. If our conjecture is right, the signs of estimates in equation (4a) should exhibit the following sign pattern:

$$\mu_1 > 0\ ,\quad \lambda_0 < 0,\quad \text{and}\quad \lambda_0 + \lambda_1 > 0\ . \tag{5}$$

The interpretation of the inequalities above are straightforward: when the economy is currently in the low-growth phase with the dummy D_t off, the lower average growth rate in the current period is captured by φ_0, and the volatility exerts adverse effect on growth via $\lambda_0 < 0$. On the other hand, when the economy is in the high-growth phase with $D_t = 1$, the average growth rate is now higher by $\mu_1 > 0$ than the φ_0 in the low-growth-phase, and increases in the volatility add to the growth rate via $\lambda_0 + \lambda_1 > 0$.

Table 2 below reports the estimation results, where various threshold levels are used to distinguish the low-growth and high-growth phases:

Table 2: GARCH-M Estimation with Business Cycle Dummy

Threshold		Estimated Coefficients		
		μ_1	λ_0	λ_1
I	$\bar{Y}$ + 0.5sd(Y)	0.129 (0.451)	−0.681 (−3.666)	1.375 (2.883)
II	$\bar{Y}$	0.015 (0.057)	−0.796 (−3.144)	1.454 (3.711)
III	$\bar{Y}$ −0.5sd(Y)	0.365 (1.062)	−0.508 (−1.838)	1.45 (2.645)
IV	$\bar{Y}$ −0.66sd(Y)	0.438 (1.262)	−0.522 (−1.388)	1.146 (2.678)

Note : $\bar{Y}$ is the sample mean of growth rates, and std(Y) is their sample standard deviation.

In the first case, where the threshold of the high−growth phase is quite high, the sign restrictions on the coefficients are all satisfied. Unlike the results in (3a), the volatility exerts significantly negative effects on the growth rate in the low−growth periods, as captured by $\hat{\lambda}_0$ =−0.681. When the economy is in the high−growth regime, however, the coefficient on the volatility becomes 1.358−0.681=0.677, which is positive and significant. Setting aside the insignificant estimate of the coefficient on the dummy, the results in the top panel of Table 2 strongly raise the possibility that the volatility−growth relation is dependent upon the business cycle phases. In the second and third cases with lower threshold levels than in the first case, the results are qualitatively the same. In the final case, we set the threshold level so that the proportion of low−growth periods is equal to that of recession periods in the NBER business cycle chronology over the sample span. Here, the effect of the high−growth dummy on the growth rate appears insignificant, but its sign is correctly estimated. Also, the positive and negative effects of volatility on growth in the high− and low−growth phases, respectively, are confirmed again, albeit the coefficient on volatility in the lower−growth phase is insignificant.

Overall, the results in Table 2 are slightly mixed in terms of the statistical significance of estimated coefficients. With regard to the signs of estimated coefficients, however, the results in Table 3 exactly square with our conjecture: the high−growth periods are associated with higher growth rate ($\mu_1>0$) and positive effects of volatility on growth (λ_0 + λ_1)>0, while higher volatility leads to lower growth ($\lambda_0<0$) in the low−growth periods.

Ⅳ. Results for the U.K. GDP

In this section, we perform a robustness check on the results obtained for the U.S. data along the international dimension. To do so, we put the real GDP series of the U.K through the same empirical procedure we did for the U.S., and examine if the business cycle dependence of the volatility−growth relation appears again.[5]

The U.K. counterpart of the GARCH−M model in (3a)−(3b) are estimated as follows:

$$\underset{(2.623)}{Y_t = 0.444} + \underset{(1.471)}{0.131\,Y_{t-1}} + \underset{(1.605)}{0.144\,Y_{t-2}} + \underset{(2.207)}{0.214\,Y_{t-3}} - \underset{(-0.893)}{0.1646 h_t^{1/2}} + \xi_t \qquad (6a')$$

$$\underset{(2.086)}{h_t = 0.009} + \underset{(31.108)}{0.145\xi_{t-1}^{2}} + \underset{(3.931)}{0.855 h_{t-1}} \qquad (6b')$$

in which the coefficient on the volatility in the mean equation is insignificant as in the U.S. case. Therefore, the GARCH−M model above fails to recover any non−trivial relation between volatility and growth.

When the high−growth dummy is included in the mean equation, however, the results in Table 3 for three levels of threshold support the sign changes of the relation across the business cycle phase:

5) We use the seasonally adjusted real GDO series of the U.K. over 1960:Q1−2009:Q3, downloaded from the ECB statistical warehouse.

Table 3: GARCH-M Estimation with Business Cycle Dummy (U.K. case)

Threshold		Estimated Coefficients		
		μ_1	λ_0	λ_1
I	$\bar{Y}$ + 0.5sd(Y)	−0.0099 (−0.342)	−0.690 (−6.783)	1.745 (6.813)
II	$\bar{Y}$	−0.133 (−0.876)	−0.747 (−3.826)	1.709 (6.810)
III	$\bar{Y}$ −0.5sd(Y)	0.536 (2.709)	−0.398 (−2.377)	0.701 (3.215)

Note : $\bar{Y}$ is the sample mean of growth rates, and std(Y) is their sample standard deviation.

In Table 3 above, the mean effect of the high-growth dummy is correctly and significantly estimated when the threshold level is as low as in the third case. Other than that, the ambivalent effects of increases in volatility on growth are sharply estimated with correct signs.

V. Conclusion

Prior research on the relationship between output volatility and growth has produced mixed results, lacking clear empirical evidence on the sign of the relationship. The aim of this paper is to provide another interpretation of the lack of clear empirical evidence. We set off with the conjecture that higher volatility exerts adverse effects on growth when the economy is in contraction and favorable effects in expansion. If the conjecture is right, then empirical investigations of data without taking this feature into account will fail to recover non-trivial relation between volatility and growth. We estimate a series of ARCH-type models with the real GDP data of the U.S. and the U.K., and find strong evidence suggesting that the volatility-growth relation is positive when the economy is in expansion, while higher volatility lowers growth rate in the contraction phase.

Although we are able to get a sense of the possibility that the volatility-growth relation depends on the business cycle phases, the results in this paper are still limited. To recapitulate, the main question to answer is "do higher *short-run* volatilities over the business cycles raise or lower *long-run* growth rate? More rigorous approach to this question requires a model that can distinguish the short-run fluctuations of the economy from its long-run trend. This is the first step of our future research.

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Abstract

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Prior research on the relationship between output volatility and growth has produced mixed results, lacking clear empirical evidence on the sign of the relationship. This paper investigates volatility-growth relationship from a different angle, with the conjecture that the sign of the volatility-growth relation changes across the business cycle phases. If the conjecture is right, then empirical investigations of data without taking this feature into account will fail to recover non-trivial relation between volatility and growth. We estimate a series of ARCH-type models with the real GDP data of the U.S. and the U.K., and find strong evidence suggesting that the volatility-growth relation is positive when the economy is in expansion, while higher volatility lowers growth rate in the contraction phase

Key words: Volatility, Growth, Business Cycles, Regime Switching
변동성, 성장, 경기변동, 국면 전환

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이름: 김장열(제1 저자)
소속: 한국외국어대학교 국제통상학과 부교수
주소: (130-79`) 서울특별시 동대문구 이문동 270
이메일: kjryoul@hufs.ac.kr

이름: 임기영
소속: 한국외국어대학교 국제통상학과 교수
주소: (130-79`) 서울특별시 동대문구 이문동 270
이메일: gylim@hufs.ac.kr