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The role of stochastic volatility and return jumps: reproducing volatility and higher moments in the KOSPI 200 returns dynamics

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Abstract This paper investigates the role of stochastic volatility and return jumps in reproducing the volatility dynamics and the shape characteristics of the Korean Composite Stock Price Index (KOSPI) 200 returns distribution. Using efficient method of moments and reprojection analysis, we find that stochastic volatility models, both with and without return jumps, capture return dynamics surprisingly well. The stochastic volatility model without return jumps, however, cannot fully reproduce the conditional kurtosis implied by the data. Return jumps successfully complement this gap. We also find that return jumps are essential in capturing the volatility smirk effects observed in short-term options.

Keywords Stochastic volatility model · Jump diffusion model · Efficient method of moments · Reprojection · Markov Chain Monte Carlo · Option pricing implications

JEL classification C14 · C15 · C52 · C53 · G13

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1 Introduction

Understanding the dynamics of equity returns is important for various financial market activities such as asset allocation, derivatives pricing, hedging, and risk management. Recent advances in the modeling of equity returns and in options pricing essentially aim to find a more realistic description of the stochastic evolution of the underlying asset price. A large proportion of these models attempt to model salient features of equity returns by introducing time-varying volatility and jump components to the Black and Scholes (1973) (BS hereafter) model.[1] Therefore, investigating the empirical adequacy of these models' specifications about return dynamics is critical to both academic researchers and financial decision makers.[2]

This paper investigates the ability of stochastic volatility jump diffusion models to capture the dynamic behavior of equity returns. Our primary focus is on the role of stochastic volatility and return jumps in reproducing heteroskedastic volatility, asymmetry (excess conditional skewness), and tail-thickness (excess conditional kurtosis) of returns distribution, which are of immediate interests in the appropriate modeling of equity returns and in options pricing.

Traditionally, the estimation of stochastic volatility jump diffusion models is challenging because they include latent state variables and there is no analytical expression available for the discrete conditional density. Recent advances in the econometric methods enable researchers to address both of these difficulties. Anderson et al. (2002) (ABL hereafter), Chernov et al. (2003) (CGGT hereafter), Gallant et al. (1997, 1999), and Gallant and Tauchen (1997) employ efficient method of moments (EMM) of Gallant and Tauchen (1996a) to estimate parameters of various return models. Eraker et al. (2003) (EJP hereafter) and Eraker (2004) estimate various jump diffusion models using Bayesian Markov Chain Monte Carlo (MCMC) method. Chacko and Viceira (2003) and Jiang and Knight (2002) perform estimation using conditional and unconditional characteristic functions of various return models, respectively. All these researches investigate the role of various factors, such as univariate- or multivariate-volatility factors and jump components, in describing the characteristics of return dynamics by exploiting information in the time series of equity returns.

We employ the EMM method like ABL, CGGT, Gallant et al. (1997, 1999), and Gallant and Tauchen (1997). The first contribution of this paper is that this paper complements the EMM estimation procedure by employing the reprojection method of Gallant and Tauchen (1998). Advantage of using the reprojection method is that it provides a convenient way for a direct comparison of the conditional density for the observed returns data that is implied by a return model with a conditional density that is computed directly from the returns data.

Relying on this method enables us to investigate how well stochastic volatility and return jumps track volatility movements, and how much conditional excess skewness and kurtosis can be reproduced by these factors, compared to those implied by the data. Answers to these questions have not been thoroughly explored in previous empirical studies and this paper provides a comprehensive picture about the models' performance in capturing the actual return dynamics. CGGT and Gallant et al. (1999) also provide the reprojection analysis. However, in CGGT and Gallant et al. (1999), the reprojection method serves a different purpose. CGGT and Gallant et al. (1999) use the reprojection

[1] See Bates (1996, 2000), Duffie et al. (2000), Eisenberg and Jarrow (1994), Heston (1993), Hull and White (1987), Scott (1987, 1997), and Wu (2006) among many others. See Johnson et al. (1997), Lee et al. (1991), and Lee et al. (2005) for the generalization of the binomial model of Cox et al. (1979).

[2] For example, Wu (2003) examines the effects of jump components on the portfolio choice problem. Baixauli and Alvarez (2006) and Huang and Lin (2004) provide excellent analyses on the application of VaR (Value-at-Risk) calculation.

method to extract the unobserved stochastic volatility factors and therefore do not directly explore the above questions.[3]

The EMM method employed here has an important distinction. Gallant and Tauchen (2005) extend the computational strategy of the EMM method by incorporating the Bayesian MCMC methods to explore the surface of the EMM criterion function, while the extant EMM procedures use derivative based hill climbing method. This extension is important for the goal of this paper because the MCMC algorithms are particularly suited for the estimation of equity return models that contain jump components.[4]

Most existing papers investigate the performance of stochastic volatility jump diffusion models in explaining the return dynamics of the developed equity market indices such as S&P 500 and FTSE 100. However, little effort has been made in examining emerging equity markets.[5] With the increased importance of emerging markets in the world economy, it is important to investigate whether empirical findings obtained for developed markets hold for emerging markets.

Another significant contribution of this paper is that this paper examines an important Asian emerging equity market: the Korea Stock Exchange (KSE). Even though the KSE is smaller than most developed countries' equity markets in terms of market capitalization, the Korean stock index options market is one of the most actively traded derivatives markets among all of the world's exchange-traded derivatives markets. The common data set used in this paper is the Korean Composite Stock Price Index (KOSPI) 200 returns. The KOSPI 200 is a market capitalization weighted index composed of 200 major stocks listed on the KSE and represents about 80% of the total market capitalization of the KSE stock market.

Among many emerging equity markets, the Korean equity market is particularly interesting for several reasons. First, as shown in Table 1, the Korean equity market is much smaller than most equity markets of the developed countries in terms of market capitalization. After successfully overcoming the 1997 Asian financial crisis, however, it showed a remarkable growth. According to the WFE (World Federation of Exchanges), the KSE is the 14th largest stock exchange in the world in 2005. Second, in terms of turnover ratio, the Korean market is highly liquid. As shown in Table 2, the turnover ratio of the Korean market is as high as that of the most developed equity markets. Third, as a result of these high growth opportunity and liquidity, a substantial proportion of the Korean equities are held by foreign investors. As shown in Table 3, foreign investors from 91 countries hold about 40% of the KSE market capitalization at the end of 2005. Fourth, the KOSPI 200 index is the underlying asset of the KOSPI 200 index options. Since introduced in July 7 1997, the KOSPI 200 index options market has shown a dramatic growth in trading volume. As shown in Table 4, the KOSPI 200 options are the most actively traded derivatives among all of the world's exchange-traded derivatives from 2000 to 2005.

Therefore, the importance of understanding the role of stochastic volatility and return jumps in explaining the KOSPI 200 returns distribution is not confined to the Korean investors. This investigation also applies to many foreign investors who hold the Korean equities in their portfolios. In addition, our investigation also delivers implication about the performance of stochastic volatility jump diffusion models in pricing the KOSPI 200 index options, which are also examined in this paper.

Several papers examine the Korean equity and options markets. Kim and Chang (1996) estimate the discrete stochastic volatility model of Harvey et al. (1994) for predicting the

[3] See Gallant and Tauchen (1998, 2002) for the general theory of reprojection and its applications.

[4] See ABL for numerical difficulties with the derivative based optimization method in the estimation of jump diffusion models.

[5] See Sect. 2 for a brief summary of recent empirical findings.

Table 1 Size and growth rate of global equity markets

	1995	1997		1999		2001		2003		2005		1995–2005
U.S. (NYSE)	5,655	8,879	36.3	11,437	28.8	11,027	−3.6	11,329	2.7	13,311	17.5	9.9 (13.9)
U.S. (Nasdaq)	1,160	1,726	32.8	5205	201.6	2,740	−47.4	2,844	3.8	3,604	26.7	19.3 (45.4)
England	1,347	1,996	32.5	2,855	43.0	2,165	−24.2	2,460	13.6	3,058	24.3	9.9 (16.4)
Germany	577	825	30.1	1,432	73.6	1,072	−25.1	1,079	0.7	1,221	13.2	11.0 (26.0)
Italy	210	345	39.1	728	111.0	527	−27.6	615	16.7	798	29.8	17.2 (25.7)
Japan	3,667	2,217	−65.4	4,555	105.5	2,265	−50.3	2,953	30.4	4,573	54.9	7.5 (35.7)
Korea	182	42	−333.3	306	628.6	195	−36.3	298	52.8	646	116.8	37.2 (77.9)
Taiwan	187	288	35.1	375	30.2	293	−21.9	379	29.4	476	25.6	12.9 (25.6)
Brazil	148	255	42.0	228	−10.6	186	−18.4	226	21.5	475	110.2	19.1 (39.1)
Mexico	91	157	42.0	154	−1.9	126	−18.2	123	−2.4	239	94.3	15.2 (33.2)
Israel	25	44	20.5	63	43.2	58	−7.9	69	19.0	123	78.3	17.5 (31.6)
Turkey	21	61	65.6	113	85.2	47	−58.4	68	44.7	162	138.2	44.5 (82.8)

This table presents the size and growth rate of stock exchanges for selected countries from 1995 to 2005. Size is the market capitalization (in billions of US dollar) and is calculated as the total number of issued shares of domestic companies multiplied by their respective closing prices at the end of each year. In each year, the first column presents the market capitalization and the second column presents the growth rate (in percentage form). The last column presents the average of year-by-year growth rates from 1995 and 2005. In the last column, the standard deviation of growth rates is given in parenthesis. Data on market capitalization exclude investment funds, warrants, ETFs, convertibles, foreign companies and include common and preferred shares. Included stock exchanges for each country are as follows. U.S.: NYSE, Nasdaq, England: London Stock Exchange, Germany: Deutsche Börse, Italy: Borsa Italiana, Japan: Tokyo Stock Exchange, Korea: KSE, Taiwan: Taiwan Stock Exchange Co., Brazil: Sao Paulo Stock Exchange, Mexico: Mexican Exchange, Israel: Tel Aviv Stock Exchange, Turkey: Istanbul Stock Exchange. Data are collected from the annual report of WFE (World Federation of Exchange, http://www.world-exchanges.org) and Korea Stock Exchange

Table 2 Turnover ratio of global equity markets

	1995	1997	1999	2001	2003	2005
U.S. (NYSE)	55.5	65.7	74.6	86.9	89.5	99.1
U.S. (Nasdaq)	228.1	237.5	303.0	359.2	280.7	250.4
England	40.5	44.0	56.7	83.8	106.6	110.1
Germany	106.3	135.2	116.9	118.3	148.1	149.4
Italy	47.8	69.1	67.7	113.4	134.8	160.0
Japan	26.0	32.9	49.4	60.0	82.6	115.3
Korea	101.0	145.5	344.9	218.7	193.1	152.3
Taiwan	199.4	407.3	288.6	206.8	190.7	131.4
Brazil	37.3	69.6	54.2	33.9	40.2	42.8
Mexico	37.0	38.8	29.2	37.8	22.4	27.2
Israel	26.9	31.6	39.3	27.3	33.0	45.1
Turkey	18.27	115.8	138.2	178.8	211.9	169.9

This table presents the average monthly turnover ratio of stock exchanges for selected countries from 1995 to 2005. Monthly turnover ratio is the ratio of trading value during each month to market capitalization at the end of each month. All figures are expressed in percentages on an annual basis. Included stock exchanges for each country are as follows. U.S.: NYSE, Nasdaq, England: London Stock Exchange, Germany: Deutsche Börse, Italy: Borsa Italiana, Japan: Tokyo Stock Exchange, Korea: Korea Stock Exchange, Taiwan: Taiwan Stock Exchange Co., Brazil: Sao Paulo Stock Exchange, Mexico: Mexican Exchange, Israel: Tel Aviv Stock Exchange, Turkey: Istanbul Stock Exchange. Data are collected from the annual report of WFE (World Federation of Exchange, http://www.world-exchanges.org) and Korea Stock Exchange

Table 3 Foreign ownership of the KSE

	1998	1999	2000	2001	2002	2003	2004	2005
Market capitalization	18.6	21.9	30.1	36.6	36.0	40.1	42.0	39.7
Number of shares	10.5	12.3	13.9	14.7	11.5	18.0	22.0	23.0
Number of countries	66	66	76	78	80	82	86	91

This table presents foreign ownership of the companies listed on the KSE. The first row presents the foreign ownership in terms of the market capitalization and the second row presents the foreign ownership in terms of the number of shares. All figures are expressed in percentages and are calculated at the end of each year. The last row presents the number of nationality of foreign investors who register with the Securities Supervisory Board (SSB). Foreign investors in Korea must register with the SSB and obtain an ID number before they can start trading stocks. Data are obtained from Korea Financial Supervisory Service (http://www.fss.or.kr)

volatility of the KOSPI 200 returns. Exploiting the time-series of the KOSPI 200 returns, Chang (1997) estimates a discrete jump model with GARCH effect and finds that there exist systematic jumps in the KOSPI 200 returns. Recently, Kim and Kim (2004, 2005) compare the performance of the option pricing models of BS, Heston (1993), and Bates (1996) in pricing the KOSPI 200 options by exploiting the cross-section of options prices. They find that stochastic volatility with return jumps model shows the best performance in terms of minimizing options pricing errors.

All these papers examine the Korean equity and options markets. However, their researches do not address the main objective of this paper. Kim and Kim (2004, 2005) do not use the information in the time-series of equity returns in their evaluation of models' performance. Kim and Chang (1996) and Chang (1997) do not investigate whether the return models estimated in their papers are able to explain the KOSPI 200 returns

Table 4 The top 5 exchange-traded derivatives by trading volume

Rank	2000		2001		2002	
1	KOSPI 200 Index Options	194	KOSPI 200 Index Options	823	KOSPI 200 Index Options	1,890
2	Euro-Bund Futures	151	Eurodollar Futures	184	Eurodollar Futures	202
3	Eurodollar Futures	108	Euro-Bund Futures	178	Euro-Bund Futures	191
4	CAC 40 Index Options	84	CAC 40 Index Options	107	E-mini S&P 500 Index Futures	116
5	T-Bond Futures	63	Euro-Bobl Futures	100	Euro-Bobl Futures	115
Rank	2003		2004		2005	
1	KOSPI 200 Index Options	2,837	KOSPI 200 Index Options	2,521	KOSPI 200 Index Options	2,535
2	Euro-Bund Futures	244	Eurodollar Futures	298	Eurodollar Futures	410
3	Eurodollar Futures	209	Euro-Bund Futures	240	Euro-Bund Futures	299
4	TIIE 28-Day Interbank Rate Futures	162	TIIE 28-Day Interbank Rate Futures	206	10-Year T-Note Futures	215
5	E-mini S&P 500 Index Futures	161	10-Year T-Note Futures	196	E-mini S&P 500 Index Futures	207

This table presents the top five exchange-traded derivatives in terms of trading volume from 2000 to 2005. In each year, the first column presents the name of derivatives and the second column presents trading volume in million of contracts. Data are collected from news releases and the annual volume survey of FIA (Futures Industry Association, http://www.futuresindustry.org)

distribution. To the best of our knowledge, this paper is the first to gauge the adequacy of stochastic volatility jump diffusion models in capturing the dynamic behavior of the KOSPI 200 returns.

We estimate four different equity return models in this paper. As a benchmark, we estimate the BS geometric Brownian motion. As a first extension of the log-normal process, the stochastic volatility (SV) model, which is consistent with the option pricing model of Heston (1993), is estimated to examine the role of stochastic volatility. In addition, the stochastic volatility model without return-volatility correlation (SV0) is considered to investigate the role of the correlation. The stochastic volatility with return jumps (SVJ) model, which is consistent with the option pricing model of Bates (1996) and Scott (1997), is estimated to investigate the role of return jumps in capturing the behavior of returns.

We also examine the option pricing implications of estimation results for the return models. To be more specific, we compare the in-sample and out-of-sample option pricing errors for the models by exploiting the KOSPI 200 options prices. We also compare the BS implied volatility (IV) curves implied by the actual option prices with those implied by the return models. None of the above papers that examine the Korean equity and options markets investigates the performance of the SV and SVJ models in explaining the volatility smile/smirk effects in the KOSPI 200 options market.

Our main results are based on the daily returns of the KOSPI 200 index from January 4, 2000 to July 29, 2005. We find several interesting results. First, the SV model performs quite well in explaining the dynamics of the KOSPI 200 returns. As summarized in Sect. 2, there is strong evidence that the SV model fails to capture the dynamics of many countries' equity indices such as S&P 500, Nasdaq 100, DJIA, Austrian ATX (Austrian Traded

Index), German DAX (Deutsche AKTIEN Index), English FTSE (Financial Times Stock Exchange) 100, and Japanese Nikkei 225. Contrary to this widespread evidence, we find that the SV model cannot be rejected at any conventional significance level by examining the KOSPI 200 returns data. Our data analysis indicates that the distributional characteristics of the KOSPI 200 returns resemble closer to the normal distribution than the S&P 500 or DJIA returns. In the SV model, as evidenced by ABL for the S&P 500 index, a negative value of the instantaneous correlation between returns and changes in volatility is important in fitting conditional negative skewness. However, our reprojection analysis suggests that the SV model cannot fully reproduce the conditional kurtosis of the data.

Second, the SVJ model shows a more striking performance. Its *P*-value of the EMM specification test is far larger than that of the SV model. Therefore, the SVJ model is the best in explaining the KOSPI 200 returns distribution. Many papers reviewed in Sect. 2 suggest that the SVJ model can capture most of the important characteristic of returns distribution. We find that the SVJ model almost completely duplicates all of the salient features of the KOSPI 200 returns in terms of the heteroskedasticity in volatility and shape deviations from conditional normality. We also find that the largest contribution of discrete jump factor lies in its ability to generate the tail-thickness of the data. This finding is consistent with the findings of ABL, EJP, CGGT, and Scott (1997).

Third, the role of return jumps is also evident in option pricing. The SVJ model performs better than the BS and SV models in fitting the cross-sectional behavior of the KOSPI 200 options prices in terms of both the in-sample and out-of-sample pricing errors. The superiority of the SVJ model is particularly remarkable for short-term options. We also find that return jumps are essential in capturing the systematic variations in the BS IV curves, i.e., the volatility smirks observed in short-term options. As summarized in Sect. 2, there is overwhelming evidence around the world equity index options markets for return jumps. Our results indicate that the KOSPI 200 options market follows the world equity index options markets with regards to this phenomenon.

The remainder of this paper is organized as follows. In Sect. 2, we present a brief summary of the recent empirical findings about the performance of various equity return models that are examined in this paper. In Sect. 3, we describe the four stock return models that are examined in this paper. Section 4 discusses the data and provides a brief summary of the EMM estimation procedure. The empirical findings from the EMM estimation and reprojection analysis are provided in Sect. 5. We explore the option pricing implications in Sect. 6. Section 7 concludes the paper.

2 Recent empirical findings

Many papers evaluate the performance of the SV and SVJ models in explaining salient features of many equity indices and option prices. First, although there are few exceptions, the SV model is found to be incapable of describing returns dynamics. Exploiting the time-series of major U.S. stock market indices such as the S&P 500, DJIA, and Nasdaq 100, ABL, CGGT, EJP, Jiang (2002), and Scott (1997) find that the SV model fails to explain the salient features of returns distributions. Similarly, using both equity returns and option prices, Chernov and Ghysels (2000), Eraker (2004), Jones (2003), and Pan (2002) conclude that the model is misspecified. Among theses papers, ABL, Eraker (2004), EJP, Jones (2003), and Scott (1997) find that unconditional/conditional higher moments, i.e., negative skewness and excess kurtosis, generated by the model fall far short of the levels observed in the actual returns data. On the other hand, CGGT, Jones (2003), and Pan

(2002) find that the model cannot fit volatility dynamics. Evidence on the stock market indices in other countries shows similar results. Based on the estimation results using daily ATX, DAX, FTSE, and Nikkei 225 index futures data, Tompkins (2000, 2001) find that the model cannot fit the unconditional excess kurtosis of the data. Similarly, using daily FTSE returns data, Daniel et al. (2005) and Jiang (2002) reject the SV model. On the contrary, Bollerslev and Zhou (2000) find that the model can explain various moment characteristics of daily Nikkei 225 returns distribution. In summary, as suggested by CGGT, the main reason for the failure of the SV model seems to be that the model cannot simultaneously explain volatility dynamics and higher moments of equity returns.

Second, as for the performance of the SVJ model, existing papers commonly find that the SVJ model performs better than the SV model in explaining returns distribution. ABL, CGGT, Jiang (2002), Pan (2002), and Scott (1997) report that the model can capture the dynamics of the S&P 500 and DJIA returns. Based on formal statistical diagnostics, ABL, CGGT, Jiang (2002), and Pan (2002) find that the model is not rejected by the S&P 500 and DJIA returns data. Jiang (2002) also finds that the model is not rejected by the FTSE returns. On the contrary, Eraker (2004) and EJP suggest that the model is partially misspecified because it cannot explain the return dynamics during extreme market stress periods such as the market crash in 1987. As suggested by ABL, CGGT, and EJP, return jumps provide additional flexibility in capturing shape deviations from unconditional/conditional normality, especially for the excess kurtosis of equity returns distribution. This additional flexibility also enables stochastic volatility to concentrate more on explaining volatility dynamics.

Third, on the option pricing performance, the SVJ model is largely found to be better than the SV model in explaining systematic variations in BS implied volatilities. Using the S&P 500 options prices and the S&P 500 futures options prices respectively, Bakshi et al. (1997) and Bates (2000) find that the SV model cannot explain the behavior of options prices across moneyness. Similarly, based on the joint estimation of the S&P 500 returns and options data, Jones (2003) and Pan (2002) find that the SV model cannot fit volatility smiles/smirks observed in short maturity options, while Pan (2002) reports that the SVJ model captures them well. Papers that examine other world markets also find similar results. Using the FTSE 100 options data, Lin et al. (2001) find that the SV model shows significant pricing errors for short maturity options. Similarly, Sepp (2003) also finds that the SV model cannot generate volatility smiles as sharp as the SVJ model for short maturity DAX option. Based on the parameter estimates using the Spanish IBEX-35 stock index data, Fiorentini, León, and Rubio (2002) find that the SV model tends to overprice out-of-the-money call options and underprice in-the-money call options.

However, Eraker (2004) reports a different result. Simultaneously exploiting both the S&P 500 returns and the S&P 500 options prices, Eraker (2004) finds that there is no discernable difference among the SV and SVJ models in the option pricing performance. Furthermore, both models are as good as other more complicated stochastic volatility jump diffusion models such as those models that incorporate jumps in volatility in capturing systematic variations in BS implied volatilities.

3 Models specifications

This section describes the stock return models considered in this paper. For convenience, we present the stochastic volatility jump diffusion model of Bates (1996) and Scott (1997) as a general representation of the models to be considered. We then discuss the strengths and weaknesses of the nested models.

Under the physical probability measure, Bates (1996) and Scott (1997) specify the stochastic evolution of the non-dividend paying stock price S as

$$\frac{\mathrm{d}S(t)}{S(t)} = (\mu - \lambda\mu_J)\mathrm{d}t + \sqrt{V(t)}\mathrm{d}w_S(t) + J(t)\mathrm{d}q(t), \tag{1}$$

$$\mathrm{d}V(t) = \kappa[\theta - V(t)]\mathrm{d}t + \sigma_V\sqrt{V(t)}\mathrm{d}w_V(t), \tag{2}$$

$$\mathrm{cov}(\mathrm{d}w_S(t), \mathrm{d}w_V(t)) = \rho\mathrm{d}t, \tag{3}$$

$$\mathrm{prob}(\mathrm{d}q(t) = 1) = \lambda\mathrm{d}t, \tag{4}$$

$$\ln(1 + J(t)) \sim N\left(\ln(1 + \mu_J) - \frac{1}{2}\delta^2, \delta^2\right), \tag{5}$$

where $V(t)$ represents the instantaneous return variance at time t, $w_S(t)$ and $w_V(t)$ are each a standard Brownian motion, λ is the frequency of jumps per year, q is a Poisson process with intensity λ, and $J(t)$ is the random percentage jump conditional on a jump occurring at time t.

3.1 Black and Scholes model

As a benchmark, the BS model is considered. The BS model assumes that the drift and diffusion coefficients are constant and that there is no return jump, i.e., $\lambda = 0$, which result in

$$\frac{\mathrm{d}S(t)}{S(t)} = \mu\mathrm{d}t + \sigma\mathrm{d}w_S(t)$$

The conditional density for stock returns implied by the BS model is the normal distribution with a constant conditional mean and variance. Therefore, the BS model can induce none of the salient features of stock return dynamics such as heteroskedatic volatility, conditional excess skewness and kurtosis.

On option pricing, it has been suggested that these simplified assumptions of the BS model may induce systematic pricing biases such as the volatility smile and smirk effects.

3.2 Stochastic volatility models

The stochastic volatility (SV) model extends the BS model by allowing the variance process $V(t)$ to be stochastic. However, it does not allow return jumps. The SV model is obtained with the restriction of $\lambda = 0$ in Eqs. (1) and (4). This model can induce instantaneous departures from normality in the stock returns distribution. First, the SV model can generate conditional excess skewness by allowing an instantaneous correlation between returns and changes in variance in Eq. (3). A negative value of ρ induces negative skewness, i.e., the leverage effect, while a positive value of ρ induces positive skewness. Second, the SV model can also generate conditional excess kurtosis. The degree of tail-thickness is driven largely by σ_V, which often is referred to as the volatility-of-volatility. Higher values of σ_V imply that the returns distribution has fatter tails.

To disentangle the role of ρ, we consider another version of the SV model with the additional restriction of $\rho = 0$. We term this specification the SV0 model. By comparing the performance of the SV and SV0 models, we can investigate the importance of ρ in capturing the conditional negative skewness.

On option pricing, Heston (1993) provides a closed-form solution for option price when the underlying price process follows the SV model. It has been suggested that higher values of σ_V imply more U-shaped BS implied volatility (IV) curves and a negative value of ρ implies downward slopping IV curves.

3.3 Stochastic volatility jump diffusion model

The full-fledged model in this paper is the stochastic volatility jump diffusion (SVJ) model of Bates (1996) and Scott (1997) described by Eqs. (1)–(5). The SVJ model allows both return jumps and stochastic volatility. By accommodating extreme movements in returns via return jumps, the model attains additional flexibility to capture departures from conditional normality. A negative mean jump size, μ_J, in Eq. (5) implies that, on average, there are more negative than positive jumps. Therefore, conditional negative skewness can be generated by two different channels, through ρ and μ_J. Similarly, the model has an additional channel that can induce conditional excess kurtosis. Higher values of δ in Eq. (5) imply that the conditional returns distribution has fatter tails.

Bates (1996) and Scott (1997) provide a closed-form solution for option price when the stock price process follows the SVJ model. The role of return jumps in option pricing is known to be more important for short maturity options than for long maturity options (e.g., Das and Sundaram, 1999). At short maturities, the IV curves implied by the SVJ model will show a more pronounced volatility smirk than those implied by the SV model when the mean jump size is negative. Similarly, the IV curves will show a more pronounced volatility smile as the variance of jump size increases.

4 Data and estimation methodology

In the first subsection, we briefly discuss the operational and institutional characteristics of the KSE, which are necessary to explain our choice of data and the distributional characteristics of the KOSPI 200 returns. In the second subsection, we discuss about our data. A brief summary of the efficient method of moments and our implementation of the method is presented in the third subsection.

4.1 Characteristics of the KSE

The KSE holds one trading session, which starts at 9:00 AM and closes at 3:00 PM on each weekday excluding Saturday. Batch auctions are used twice a day to determine the opening price and the closing price. There are no trades during the last 10 min of each day, when orders are collected for the closing batch auction at 3:00 PM. Trading prices during the rest of the trading hours are determined by continuous auction. The KSE does not have designated market makers. Both market and limit orders from buyers and sellers meet via the Automated Trading System (ATS).

There is a price change limit system in the KSE. The KOSPI 200 index cannot fall or rise by more than 15% from the closing price of the previous trading day. Before November 25, 1996, this price change limit was 6%. It then increased to 8% on November 25, 1996, to 12% on March 2, 1998, and finally to 15% on December 7, 1998. Therefore, the largest market crash that can occur in the KSE is bounded above at 15%.

4.2 Data analysis

Our empirical results are based on daily returns of the KOSPI 200 index from January 4, 2000 to July 29, 2005. The sample size is 1,369 observations. Daily returns are defined as 100· (log P_t −log P_{t-1}), where P_t is the daily KOSPI 200 index price. Daily KOSPI 200 index prices are obtained as follows.

Using minute-by-minute transaction prices of the KOSPI 200 index obtained from the KSE, we sample the KOSPI 200 index price at 2:50 PM every day. This sampling time is chosen for the investigation of the option pricing implications of our EMM estimation results, which is presented in Sect. 6. As discussed above, from 2:50 PM to 3:00 PM, there is no transaction of stocks that compose the KOSPI 200 index. However, the KOSPI 200 options are traded during these 10 min. For this reason, we cannot use the KOSPI 200 options prices after 2:50 PM. Therefore, although the distributional characteristics of two daily returns data sets, which are constructed by using the daily KOSPI 200 index prices sampled at 2:50 PM and 3:00 PM, respectively, are quite similar, it is more appropriate to use the KOSPI 200 index prices sampled at 2:50 PM.[6]

It is widely recognized that stock returns exhibit time-varying volatility, negative skewness, and excess kurtosis.[7] As discussed in the previous section, these features of stock returns cannot be accommodated by the BS log-normal process. Panel A and Panel B of Fig. 1 plot daily price level and return of the KOSPI 200 index.[8] A basic observation of the KOSPI 200 returns is that large (small) price changes tend to be followed by other large (small) price changes. For example, the range for returns is wider in year 2000 and much narrower in year 2005. This characteristic is confirmed by the historical volatility. Figure 2 shows the historical volatility of the KOSPI 200 returns obtained by the BEKK-GARCH(1,1) model proposed by Engle and Kroner (1995) and the moving average of sample standard deviations. As shown in the plot, periods of high volatility can be distinguished from periods of low volatility. We find that the persistence parameter in the GARCH model is close to one and highly significant. Therefore, it is clear that the KOSPI 200 returns show pronounced time-varying volatility. This observation suggests that stochastic volatility is an important ingredient in the KOSPI 200 returns.

At the same time, the KOSPI 200 returns are not normally distributed. Panel A of Table 5 presents the summary statistics of the KOSPI 200 returns from January 4, 2000 to July 29, 2005. The marginal distribution of the KOSPI 200 returns is negatively skewed and leptokurtic; its kurtosis exceeds 3.0, the kurtosis value of the normal distribution. The statistic in the last column of Table 5 is the Jarque-Bera (JB) statistic for detecting departures of data from normality. The JB statistic suggests that we have to reject normality at any conventional significance level.

For more investigation about these properties, Fig. 3 shows a histogram of the normalized daily returns of the KOSPI 200 index and the standard normal density. It

[6] We find that there is no discernable difference in the marginal distribution between these two data sets. Daily returns constructed by using the price at 2:50 PM are slightly more negatively skewed and leptokurtic.

[7] For example, recently, Chiang and Doong (2001) and Selçuk (2005) examine the volatility dynamics of many emerging stock market indices and find that there is a considerable volatility persistence in these markets. Cappiello et al. (2003) estimate unconditional moments of FTSE All-World Indices for 21 countries. Using weekly returns data, they find that all the equity indices are leptokurtic and the returns for 19 countries are negatively skewed.

[8] Our main results are obtained by using daily returns of the KOSPI 200 index from January 4, 2000 to July 29, 2005. Daily returns from January 4, 1997 to December 28, 1999 are reserved for the analysis presented in Subsect. 5.4.

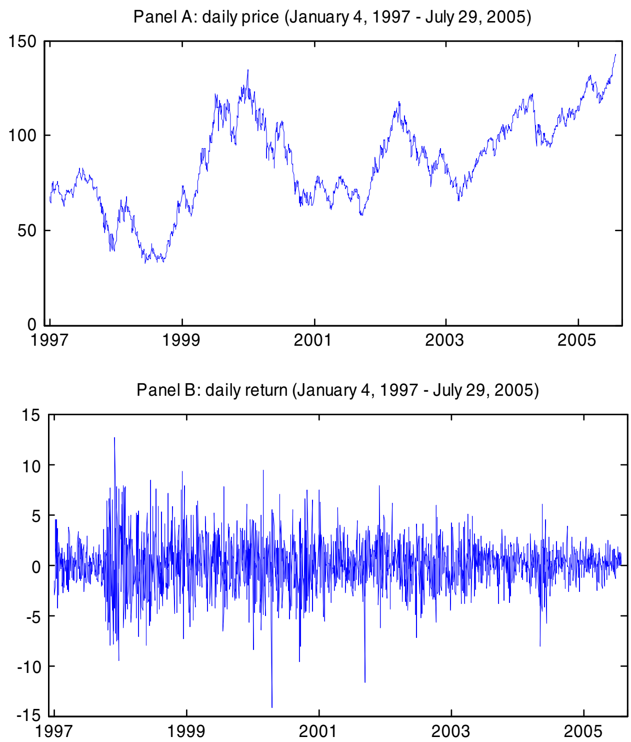

Fig. 1 KOSPI 200 index price and return. *Panel A and Panel B* show daily KOSPI 200 index price and return from January 4, 1997 to July 29, 2005. Daily returns are defined as 100· (log P_t −log P_{t-1}), where P_t is the KOSPI 200 index price

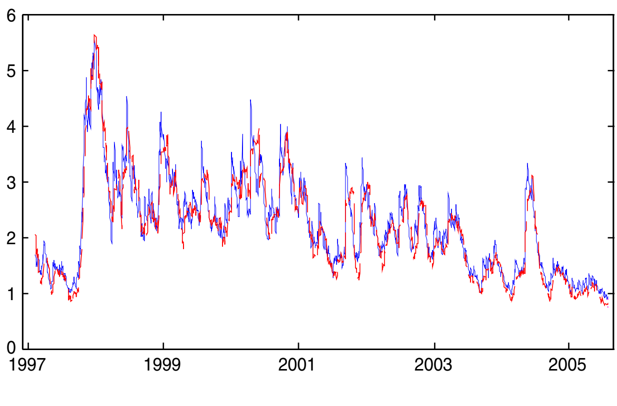

Fig. 2 Historical volatility of daily KOSPI 200 returns. This figure shows the historical volatility of daily returns of the KOSPI 200 index implied by the GARCH(1,1) model (sold line) and by the moving average of sample standard deviations (dashed line) from January 4, 1997 to July 29, 2005. Daily returns are defined as 100· (log P_t −log P_{t-1}), where P_t is the KOSPI 200 index price. The first 30 observations are reserved to form lagged information

Table 5 Summary statistics for daily KOSPI 200 returns

Mean	Std. Dev.	Skewness	Kurtosis	Auto.Corr.	Min	Max	JB-statistics
Panel A : KOSPI 200 returns (January 4, 2000–July 29, 2005)							
0.0045	2.0683	−0.4348	6.8394	0.0428	−14.1321	9.4402	871.4058
Panel B : KOSPI 200 returns (January 4, 1997–July 29, 2005)							
0.0332	2.3462	−0.0587	5.5901	0.0936	−14.1321	12.6363	614.3902

Panel A presents summary statistics for daily returns of the KOSPI 200 index from January 4, 2000 to July 29, 2005 (1,369 observations). *Panel B* presents summary statistics for daily returns of the KOSPI 200 index from January 4, 1997 to July 29, 2005 (2,202 observations). Daily returns are defined as 100· (log P_t −log P_{t-1}), where P_t is the KOSPI 200 index price. JB-statistics refer to the Jarque-Berra test statistics. All figures are expressed in percentages on a daily basis

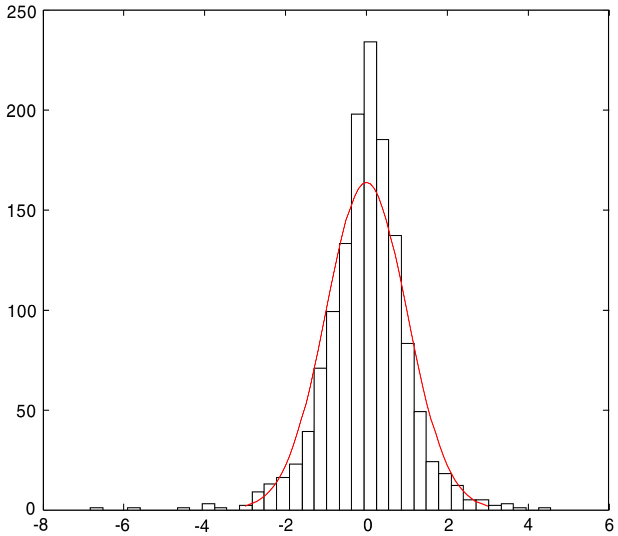

Fig. 3 Histogram of daily KOSPI 200 returns. This figure shows a histogram for normalized daily returns of the KOSPI 200 index and an approximation by the standard normal density. Daily returns are defined as 100· (log P_t −log P_{t-1}), where P_t is the daily KOSPI 200 index price from January 4, 2000 to July 29, 2005

indicates that the marginal distribution of the KOSPI 200 returns is peaked with fatter tails than the normal density with a bit of asymmetry. There are observations above four standard deviations, which is extremely unlikely in the normal distribution. Especially, the largest single-day movement of the KOSPI 200 index lies beyond six standard deviations.

These properties of higher moments cannot be captured by the BS model. Therefore, we can expect that stochastic volatility and return jumps need to be incorporated into the BS model in order to capture the shape characteristic of the KOSPI 200 returns distribution. As discussed in the next section, many of these characteristics of the KOSPI 200 returns are reflected in the conditional seminonparametric density of Gallant and Tauchen (2005).

For more investigation about the characteristics of the KOSPI 200 returns, we discuss some basic summary statistics of the KOSPI 200 returns together with those of the S&P

500 returns. For this purpose, we refer to ABL. They report that the sample mean, standard deviation, skewness, and kurtosis of the daily S&P 500 returns from January 3, 1980 to December 31, 1996 are 0.0453, 0.9618, −3.3390, and 83.4004, respectively.[9] First, the KOSPI 200 index is much more volatile than the S&P 500 index. The standard deviation of the KOSPI 200 returns is about 2.07% per day, more than double the standard deviation of the S&P 500 returns of 0.96%. Second, and more importantly, the degree of negative skewness and excess kurtosis of the KOSPI 200 returns is far less than that of the S&P 500 returns. Third, the largest movement in a single day of the KOSPI 200 index is much smaller than in the S&P 500 index.

In summary, the KOSPI 200 has high volatility, which exhibits time-varying property. However, it can move only within a bounded range because of the price change limit system in the KSE. The largest market movement in a single day that can occur in the KSE is bounded above at 15%. Consequently, the KOSPI 200 returns cannot show an extreme degree of negative skewness and excess kurtosis like those observed in the S&P 500 or DJIA returns.

As will be reported, our empirical results suggest that both the SV and SVJ models perform quite well in capturing the dynamics of the KOSPI 200 returns. However, as discussed in Sect. 2, many extant papers report that the SV model cannot explain many of the salient features of equity returns. Therefore, it is conjectured that the distributional characteristics of the KOSPI 200 returns, which resemble closer to the normal distribution than the S&P 500 returns, play an important role.

4.3 Efficient method of moments

Our focus is to investigate whether the restrictions of the return models can be justified by the discretely observed returns data. This task requires analytical expressions for the discrete conditional density implied by the continuous-time return models. However, the estimation of the SV and SVJ models is challenging because they include latent state variables and there is no analytical expression for the conditional density available. Consequently, an exact maximum likelihood estimation is not feasible. In this paper, we employ the efficient method of moments (EMM) of Gallant and Tauchen (1996a) to estimate the parameters of the stock return models. Recently, ABL, Chernov and Ghysels (2000), CGGT, Gallan et al. (1997, 1999), and Gallant and Tauchen (1997) have applied the EMM method to estimate the parameters of the SV and SVJ models for the S&P 500 and DJIA returns.

To begin with, we briefly summarize the method, then we describe our implementation.[10] Let $p\,(y_t \mid x_{t-1}, \mathrm{P})$ represent a conditional density for a discrete stationary time series implied by a diffusion model, where y_t denotes the current observation of the series, x_{t-1} denotes the lagged observations, and P is a vector of unknown parameters of the diffusion model. In this paper, y_t is the daily KOSPI 200 return and P is a vector of the parameters of the candidate stock return model. As noted above, however, no analytic expression of $p(y_t \mid x_{t-1}, \mathrm{P})$ is available for the SV or the SVJ model. An extension to the maximum likelihood estimation is to simulate the evolution of the factors and match the moments, which is known as simulated method of moments. Gallant and Tauchen (1996a) suggest using the score vector from an auxiliary model as the vector of moments in the simulated method of moments. Let $f\,(y_t \mid x_{t-1},\Theta)$ denote an auxiliary model for discretely sampled data where Θ denotes a vector of the parameters of the auxiliary model. If this auxiliary model, which is

[9] See their Table 1 on page 1250. All figures are expressed on a daily basis in percentage form.

[10] For details of the EMM, see Gallant and Tauchen (1996a, 2001, 2002).

termed the score generator, is fitted by quasi-maximum likelihood to get an estimate$\tilde{\Theta}_n$, then the average of the score vector over the data $\{\tilde{y}_t, \tilde{x}_{t-1}\}_{t=1}^{n}$ satisfies

$$\frac{1}{n}\sum_{t=1}^{n}\frac{\partial}{\partial\Theta}\log f\left(\tilde{y}_t|\tilde{x}_{t-1},\tilde{\Theta}_n\right)=0, \tag{6}$$

because Eq. (6) is the first order condition of the optimization problem. The score functions also can be evaluated over a long simulated data $\{\hat{y}_t(\mathrm{P}), \hat{x}_{t-1}(\mathrm{P})\}_{t=1}^{N}$, which is generated by the stock return model for a candidate value of P. If the stock return model is correctly specified, then the average of the score vector over $\{\hat{y}_t(\mathrm{P}_0), \hat{x}_{t-1}(\mathrm{P}_0)\}_{t=1}^{N}$ would be close to zero where P_0 denotes the true but unknown value of P. Based on this idea, the moment conditions for the EMM estimator are defined as

$$m(\mathrm{P},\tilde{\Theta}_n)=\frac{1}{N}\sum_{t=1}^{N}\frac{\partial}{\partial\Theta}\log f\left(\hat{y}_t(\mathrm{P})|\hat{x}_{t-1}(\mathrm{P}),\tilde{\Theta}_n\right), \tag{7}$$

with N large enough that the Monte Carlo simulation error is negligible. Then, the EMM estimator is obtained by minimizing the EMM criterion function, i.e.,

$$\hat{\mathrm{P}}_n=\arg\min\nolimits_{\mathrm{P}} m'(\mathrm{P},\tilde{\Theta}_n)\left(\tilde{I}_n\right)^{-1}m(\mathrm{P},\tilde{\Theta}_n), \tag{8}$$

where $\left(\tilde{I}_n\right)^{-1}$ denotes the quasi-information matrix from quasi-maximum likelihood estimation of Θ. If $p(y_t \mid x_{t-1}, \mathrm{P})$ is a correct model, then the statistic

$$n\, m\left(\hat{\mathrm{P}}_n,\tilde{\Theta}_n\right)'\left(\tilde{I}_n\right)^{-1}m\left(\hat{\mathrm{P}}_n,\tilde{\Theta}_n\right), \tag{9}$$

is asymptotically chi-squared on $l_\Theta - l_\mathrm{P}$ degrees of freedom where l_Θ and l_P are, respectively, the lengths of parameter vectors Θ and P. Details of the EMM implementation are presented in the Appendix A.

5 Empirical results

The first subsection reports the estimation of the SNP score generator. We report and discuss the EMM estimation results in the second subsection. The third subsection reports reprojection analysis. The final subsection reports the SNP and EMM estimation results over an extended sample period.

5.1 Estimation of the SNP density

The first step of the EMM procedure is fitting the score generator. We use the SNP score generator for the daily KOSPI 200 returns, defined as 100· (log P_t −log P_{t-1}), where P_t is the daily KOSPI 200 index price.[11] In the SNP procedure, the first 38 observations are reserved to form the lagged information. Following the suggestion of Gallant and Tauchen

[11] Therefore, we simulate log stock price in the second step of the EMM procedure.

(2005), we apply a spline transformation to the raw data to improve the stability of the quasi-maximum likelihood estimation.

Following Gallant and Tauchen (2005), we select the tuning parameters of the SNP density, { L_u , L_r , L_p , K_z , max K_z , K_x} by moving along an upward density expansion path using the Bayes information criterion (BIC)

$$\mathrm{BIC} = s_n(\tilde{\Theta}_n) + \frac{l_\Theta}{2n}\log(n),$$

$$s_n\left(\tilde{\Theta}_n\right) = -\frac{1}{n}\sum_{t=1}^{n}\log f_K\left(\tilde{y}_t\middle|\tilde{x}_{t-1},\tilde{\Theta}_n\right),$$

to guide the search. The SNP densities with small BIC values are preferred.

Table 6 presents the choice of SNP density and the BIC value. To be conservative, we also expand K_z with L_r = { 2, 3, 4, 5, 6, 7, 8}.[12] Our BIC preferred SNP density is described by {L_u, L_r , L_p , K_z , max K_z , K_x } = { 1, 9, 1, 6, 0, 0 }. With this specification, AR(1) process describes the conditional mean dynamics and ARCH(9) process determines the conditional volatility dynamics of the KOSPI 200 returns. We need a sixth-order Hermite polynomial in standardized innovation to capture the shape deviations from conditional normality. K_x = 0 implies that it is unnecessary to include the lagged returns in modeling the coefficients of the Hermite polynomial, and therefore the shape of the preferred SNP density is homogeneous. The number of the parameters of our preferred SNP density is 18. Our SNP density is akin to the semiparametric ARCH class of densities proposed by Engle and Gonzales-Rivera (1991).

5.2 Parameter estimates and specification tests

The EMM estimation results are summarized in Table 7, which presents parameter estimates and specification tests for each of the four stock return models. The last row shows the *P*-value of the goodness-of-fit test for each of the models. To get additional insight into the performance of the models, Table 8 presents the *t*-ratios for the scores of the best model fit with respect to the SNP parameters. With these diagnostic *t*-ratios, we can analyze the strengths and weaknesses of the different model specifications. Different elements of the score correspond to different characteristics of the data. If a given stock return model is capable of matching a particular score, then the *t*-ratio for that score should not be large. A *t*-ratio above 2.0 in magnitude indicates that the model fails to fit the corresponding score.[13] To begin with, we discuss the results of the BS model, then investigate the effects of stochastic volatility and return jumps.

5.2.1 BS model

The first column of Table 7 represents the parameter estimates and the goodness-of-fit test for the BS model. The model is rejected at any conventional significance level. The first

[12] The results are not reported here but are available upon request.

[13] Strictly speaking, we report the quasi *t*-ratios, which are suggested by Gallant and Long (1997) and Tauchen (1997) and are commonly used in the papers that employ the EMM method. See, for example, CGGT, Gallant et al. (1997), and Gallant and Tauchen (1996b, 1997, 1998) among many others.

Table 6 Choice of SNP density

L_u	L_r	L_p	K_z	max K_z	K_x	l_θ	$s_n(\tilde{\Theta}_n)$	BIC
1	0	1	0	0	0	3	1.36155	1.36947
2	0	1	0	0	0	4	1.36021	1.37077
3	0	1	0	0	0	5	1.36019	1.37338
1	1	1	0	0	0	4	1.35116	1.36171
1	2	1	0	0	0	5	1.32411	1.33730
1	3	1	0	0	0	6	1.32322	1.33905
1	4	1	0	0	0	7	1.31213	1.33060
1	5	1	0	0	0	8	1.30459	1.32569
1	6	1	0	0	0	9	1.29008	1.31383
1	7	1	0	0	0	10	1.28523	1.31161
1	8	1	0	0	0	11	1.27734	1.30636
1	9	1	0	0	0	12	1.29970	1.29971
1	10	1	0	0	0	13	1.26773	1.30203
1	11	1	0	0	0	14	1.26773	1.30466
1	9	1	4	0	0	16	1.24730	1.28950
1	9	1	5	0	0	17	1.24589	1.29074
1	**9**	**1**	**6**	**0**	**0**	**18**	**1.24159**	**1.28908**
1	9	1	7	0	0	19	1.24026	1.29038
1	9	1	8	0	0	20	1.24025	1.29301
1	9	1	6	6	1	25	1.23597	1.30192
1	9	1	6	5	1	24	1.23614	1.29945
1	9	1	6	4	1	23	1.23713	1.29780
1	9	1	6	3	1	22	1.23820	1.29623
1	9	1	6	2	1	21	1.23827	1.29367
1	9	1	6	1	1	20	1.23852	1.29128

This table presents upward expansion path of the SNP density and the BIC criterion. The number of lags L_p in the x_{t-1} part of the polynomial is inoperative if $K_x = 0$ and set to 1 by convention. The BIC preferred SNP density is 191600. The last two columns report the minimized objective function and the BIC criterion

column of Table 8 represents the t-ratios for the significance of the scores implied by the BS model. The t-ratios for the scores with respect to the ARCH terms suggest that the model has considerable difficulty in capturing the volatility dynamics of the data. Heteroskedastic volatility modeled by ARCH is not admissible in the BS model specification because it can generate only a constant return volatility.

Similarly, the scores for the Hermite terms suggest that the model cannot capture the shape characteristics of the density. All of the t-ratios for the scores with respect to the odd powers of the Hermite terms are below 2.0 in magnitude. However, these t-ratios should not be interpreted as evidence of the model's success in capturing the conditional skewness of the data. Gallant and Tauchen (1996b) note that although the odd powers of the Hermite terms tend to control conditional skewness and the even powers tend to control conditional kurtosis, the association between the degree and the features of the density is not quite exact because the polynomial is squared as described in the Appendix B. In the following

Table 7 Parameter estimates and specification tests

Parameter	BS	SV0	SV	SVJ
μ	0.1876 (0.0018)	0.1656 (0.0032)	0.1257 (0.0079)	0.1000 (0.0028)
σ	0.2884 (0.0001)			
κ		5.7526 (0.0591)	4.6494 (0.0612)	4.4620 (0.0469)
θ		0.0482 (0.0000)	0.0631 (0.0002)	0.0626 (0.0001)
σ_V		0.3354 (0.0028)	0.5137 (0.0018)	0.5072 (0.0012)
ρ			−0.4095 (0.0089)	−0.4034 (0.0043)
λ				7.0940 (0.5256)
μ_J				−0.0032 (0.0003)
δ				0.0258 (0.0003)
χ^2	35.9810	21.0050	11.9810	5.3283
df	16	14	13	10
P-value	0.0029	0.1015	0.5292	0.8682

This table presents parameter estimates and goodness-of-fit tests for the stock return models. The model and parameters are described in Sect. 3. The parameter estimates are expressed in decimal form on an annual basis. Standard errors are given in parentheses. The last three rows report χ^2 statistics for the goodness-of-fit of the models, degrees of freedom, and corresponding *P*-value

Table 8 Diagnostic *t*-ratios

		BS	SV0	SV	SVJ
AR	b_0	1.4138	−1.9321	−1.0989	−1.1556
AR	B	−0.2079	0.1991	−0.1964	−0.2689
ARCH	R_0	2.9122	−1.5835	−1.1647	−0.7984
ARCH	P_1	3.5918	−0.2652	−0.3704	0.1165
ARCH	P_2	−0.6370	1.7235	0.5686	0.5394
ARCH	P_3	−2.1373	1.7972	0.5059	1.0615
ARCH	P_4	−1.6180	0.1716	0.5692	0.7858
ARCH	P_5	0.4279	1.3528	0.2528	0.2065
ARCH	P_6	0.6388	−1.8840	−1.7188	−0.4605
ARCH	P_7	1.6467	−1.6006	0.2403	−0.8166
ARCH	P_8	1.0253	1.9092	−0.5631	−0.2025
ARCH	P_9	0.0932	2.1524	0.9878	1.1974
Hermite	a_{01}	1.2287	−1.3300	−1.2878	−1.1219
Hermite	a_{02}	2.2439	−2.1290	−1.1766	−0.9873
Hermite	a_{03}	0.6382	1.5850	1.0018	1.1232
Hermite	a_{04}	−2.8273	−0.5796	0.1807	0.4326
Hermite	a_{05}	0.1225	−1.5219	−1.1080	−0.7987
Hermite	a_{06}	1.9149	2.2058	1.6738	0.2423

This table presents *t*-ratio diagnostics for EMM scores evaluated for the SNP score generator 191600. The *t*-ratios are the test statistics of the null hypothesis that the scores with respect to the parameters of the SNP density are equal to zero reprojection analysis, we confirm that the BS model can capture neither the negative skewness nor the excess kurtosis of the data.

5.2.2 SV0 model

The second column of Table 7 represents the effects of the stochastic volatility factor. Although the SV0 specification does not allow instantaneous correlation between returns and changes in volatility, its performance greatly improves that of the BS model. Interestingly, the *P*-value is above 10%. Therefore, allowing the stock return volatility to be stochastic is an important factor to capture the dynamics of the KOSPI 200 returns. ABL report that the SV0 model is overwhelmingly rejected. Furthermore, they report that the performance of the SV0 model is nearly indistinguishable from that of the BS model when the models are applied to the S&P 500 returns. Our result should not be surprising because the deviations from normality of the KOSPI 200 returns are far smaller than those of the S&P 500 returns.

As shown in the second column of Table 8, the diagnostic *t*-ratios for the SV0 model suggest that the improvement comes mostly from the model's ability to fit the conditional volatility dynamics. The SV0 model has only one *t*-ratio greater than 2.0 for the scores with respect to the ARCH terms. However, the *t*-ratios of the Hermite terms suggest that the model still has difficulty in capturing the shape characteristics of the conditional density. In theory, the SV0 model can generate excess kurtosis via σ_V. Our estimate for σ_V is 0.3354 and significant. The model, however, cannot generate negative skewness because the two shocks to returns and changes in volatility are independent. Therefore, we cannot figure out exactly where the problems lie. In the following reprojection analysis, we find that the model can generate only a small amount of the excess kurtosis and cannot capture the negative skewness of the data.

5.2.3 SV model

The third column of Table 7 represents the estimation results of the SV model. The specification test of the SV model is interesting. The *P*-value for the model is 0.5292. Furthermore, all of its *t*-ratios are below 2.0. The key feature of the SV model compared to the SV0 model is allowing returns to be correlated with changes in volatility, thereby accommodating the negative skewness of the data. However, comparing the estimates of the parameters of the SV model to those of the SV0 model reveals that the effects of relaxing the correlation parameter ρ to be free are extensive. First, the estimate of ρ is -0.4095 and highly significant. Second, the estimates of the remaining volatility factor parameters, i.e., κ, θ, and σ_V, show dramatic changes compared to those of the SV0 model. The estimate of κ drops sharply from 5.7526 (in the SV0 model) to 4.6494 (in the SV model). On the other hand, both of the estimates of (θ , σ_V) are increased dramatically from (0.0482, 0.3354) to (0.0631, 0.5137). Therefore, the unobserved volatility factor becomes more persistent and volatile with higher long-run average. ABL report similar changes in the configuration of their parameter estimates for the SV and SV0 models to those observed here.[14]

As shown in the third column of Table 8, all of the *t*-ratios for the SV model are below 2.0. Therefore, the SV model is able to adequately capture all of the relevant features of the data. The diagnostic *t*-ratios show the effects of differences in the parameter estimates in

[14] See their parameter estimates for SV_2, $\rho = 0$ and SV_2, $\rho \neq 0$ in their Tables 3 and 6.

the SV and SV0 models. First, the t-ratios for the scores with respect to the ARCH terms are dramatically improved over those of the SV0 model. A reason for this improvement can be found in the change in the estimated κ. CGGT model the conditional volatility by GARCH in their preferred SNP density for the DJIA returns. They suggest that the lower the value of κ, the better GARCH volatility is captured. Our results indicate that a decrease in the value of κ seems to be also advantageous in capturing the ARCH effects. Another reason for this improvement is the increase in the value of θ. In the following reprojection analysis, we find that the SV0 model has considerable difficulty in reproducing the level of the conditional volatility, while the SV model fits better. Second, and as expected, all of the t-ratios for the scores of the Hermite terms are significantly improved over those of the SV0 model. Therefore, the SV model performs better in capturing both the excess kurtosis and the negative skewness of the data. In the SV model, conditional skewness and kurtosis are largely induced by ρ and σ_V, respectively. As noted above, our estimate of ρ is -0.4095 and highly significant. The t-ratios suggest that this value can reasonably describe the negative skewness of the data. The SV model has a larger value of σ_V than the SV0 model. Therefore, the SV model is able to capture the excess kurtosis of the data better than the SV0 model.

5.2.4 SVJ model

The final column of Table 7 represents the estimates and the goodness-of-fit test for the SVJ model. Surprisingly, the P-value for the SVJ model is 0.8682, and all of the t-ratios are below 2.0. These results suggest that the model can almost completely capture all features of the data, which are characterized by our preferred SNP density. The SVJ model also strongly outperforms the SV model in overall performance. Therefore, return jump factor is another critical ingredient for the dynamics of the KOSPI 200 returns when it is coupled with the stochastic volatility factor.

The estimates of the jump parameters are of interest. The value of λ is 7.0940, which indicates that jumps occur about seven times per year. Our estimate of μ_J is -0.32% and significant, which is consistent with the negative skewness of the data. This means that the jump factor contributes to fitting the negative skewness of the data. The value of the last jump parameter δ is about 2.6% and highly significant, so most jumps fall within the ±5.2% range. The estimates of the volatility factor parameters κ, θ, σ_V, and ρ show a mild decrease compared to those of the SV model. Therefore, incorporating return jumps has the effect of reducing the demands on stochastic volatility. This finding is consistent with CGGT and EJP.

As shown in the last column of Table 8, the t-ratios for the scores of the Hermite terms suggest that the SVJ model outperforms the SV model in capturing the shape characteristics of the density. Especially, the t-ratio for the score of the sixth Hermite term is dramatically improved, from 1.6738 (in the SV model) to 0.2423 (in the SVJ model). Clearly, jump factor contributes to capturing the tail behavior of the data. There is a mild improvement in the t-ratios for the scores of the ARCH terms. For example, among the scores of the ARCH terms, the largest t-ratio is 1.1974 in the SVJ model, but -1.7188 in the SV model. CGGT also find that return jumps induce a significant improvement in fitting the volatility characteristics of the DJIA returns.[15] It seems that the interpretation of CGGT can also be applied to our results. That is, return jumps provide additional flexibility in capturing the shape deviations from conditional normality of the data, thereby enabling stochastic volatility to concentrate more on capturing the volatility dynamics.

[15] See their t-ratios for the scores of the GARCH terms for models termed AFF1V, AFF1V-J0, and AFF1V-J presented in their Table 5.

5.3 Reprojection

The reprojection method of Gallant and Tauchen (1998) provides additional diagnostics for the adequacy of the return models. This method characterizes the dynamics of the stock returns conditional on their lags. Our preferred SNP density obtained in Subsect. 5.1, i.e., the projected conditional density, represents the unrestricted conditional density for the observed KOSPI 200 returns data. The reprojection method provides a way to get the conditional density for the observed KOSPI 200 returns data implied by the return models, i.e., the reprojected conditional density.

Let $p\ (y_t \mid x_{t-1})$ denote the conditional density for the data implied by the candidate return model, where y_t denotes the contemporaneous stock return and x_{t-1} denotes the lagged stock returns. Because no analytical expression of the conditional density implied by the SV or the SVJ model is known, we cannot estimate it by $\hat{p}(y_t|x_{t-1}) = p(y_t|x_{t-1}, \hat{\mathrm{P}}_n)$, where $\hat{\mathrm{P}}_n$ denotes the estimated model parameters presented in Table 7. Gallant and Tauchen (1998) suggest using $f_K(\hat{y}_t|\hat{x}_{t-1}, \hat{\Theta})$ as an approximation of $\hat{p}(y_t|x_{t-1})$, where $\{\hat{y}_t, \hat{x}_{t-1}\}_{t=1}^{N}$ is a long simulated data generated by $\hat{\mathrm{P}}_n$ and $f_K(\hat{y}_t|\hat{x}_{t-1}, \hat{\Theta})$ is an SNP density with the K-dimensional parameter vector $\hat{\Theta}$. Gallant and Long (1997) show that $f_K(\hat{y}_t|\hat{x}_{t-1}, \hat{\Theta})$ converges to $\hat{p}(y_t|x_{t-1})$ as K goes to infinity.[16] We estimate the SNP density for the simulated stock returns using the specification of 191600, which was used to characterize the conditional density for the observed stock returns.

If the stock return model is correctly specified, then the reprojected conditional density would be very close to the projected conditional density. Therefore, the reprojected conditional density can be used to assess the performance of the model in capturing the particular moments implied by the data. In this paper, we compare the one-step-ahead conditional volatilities and the shape deviations from conditional normality implied by the stock return models to those implied by the data.

Figures 4 and 5 compare the conditional volatilities implied by the data to those implied by the BS, SV0, SV, and SVJ models. As expected, Panel B of Fig. 4 indicates that the BS model fails to capture the volatility dynamics implied by the data that is depicted in Panel A of Fig. 4. The conditional volatility implied by the BS model specification is nearly constant.

Panel C of Fig. 4 shows the conditional volatility implied by the SV0 model. The SV0 model seems to be capable of reproducing the general shape of the conditional volatility of the data. However, the plot suggests that the SV0 model has considerable difficulty in capturing the level of conditional volatility. This problem becomes more evident during periods of high volatility.

As shown in Panel B of Fig. 5, the SV model duplicates the projected volatility fairly well. The shape of the reprojected volatility is quite close to the projected volatility. The SV model is also capable of reproducing the volatility levels during periods of average and low volatility. However, the plot suggests that the model has mild difficulty in reproducing the volatilities as high as the data in some peaks of the plot.

Panel C of Fig. 5 depicts the conditional volatility implied by the SVJ model. The SVJ model reproduces the volatility dynamics better than the SV model. The SVJ model can track almost every detailed movement along the volatility path. In particular, the SVJ model is better than the SV model at duplicating the volatility level in peaks of high volatility periods. Therefore, the SVJ model is the best at tracking the volatility of the data. However, it should be noted that the SV model also does a good job of reproducing the conditional volatility.

[16] Convergence is in terms of the Sobolev norm specified by Gallant and Long (1997).

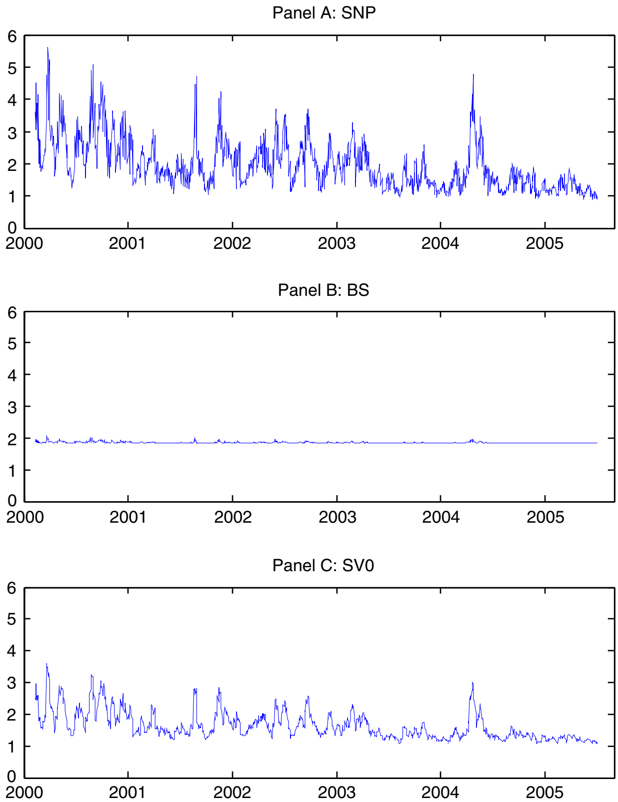

Fig. 4 Projected and reprojected conditional volatility: Projection, BS, and SV0. *Panel A* presents the conditional volatility implied by the SNP fit to the data (projected conditional volatility), *Panel B* presents the conditional volatility implied by the BS model, and *Panel C* presents the conditional volatility implied by the SV0 model. The models are described in Sect. 3

Figure 6 plots the one-step-ahead conditional densities implied by the observed data and the models. In the plots, all the lagged stock returns are set to the unconditional mean of the data. Within each plot, the solid line is the projected or reprojected conditional density and the dashed line is the normal density with the same mean and variance. These plots show different models' performance in capturing the shape deviations from conditional normality implied by the data. The unrestricted conditional density, which is the projected conditional density, is leptokurtic and negatively skewed.

Figure 6 reveals some important differences across the four return models. The four stock return models show remarkably different abilities to capture the shape deviations from conditional normality implied by the data. As shown in Panel B of Fig. 6, the conditional density implied by the BS model is the normal density. Therefore, the BS model is able to capture neither the excess kurtosis nor the negative skewness implied by the data that is depicted in Panel A of Fig. 6.[17]

[17] The reprojected conditional densities for the BS and SV0 model have wider domains than the projected and other reprojected densities. These two models overestimate the conditional variance when conditioned by the sample mean of the data. However, this result does not alter the shape of the density.

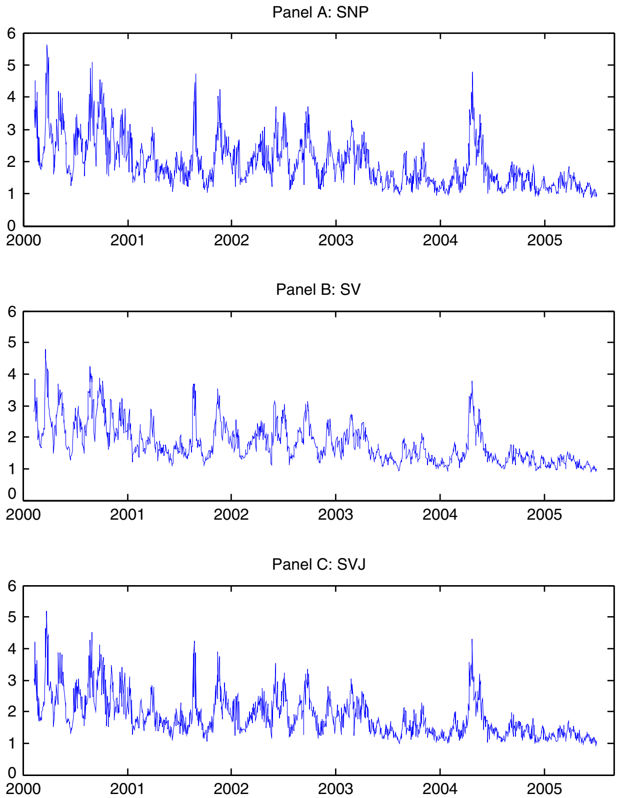

Fig. 5 Projected and reprojected conditional volatility: Projection, SV, and SVJ *Panel A* presents the conditional volatility implied by the SNP fit to the data (projected conditional volatility), *Panel B* presents the conditional volatility implied by the SV model, and *Panel C* presents the conditional volatility implied by the SVJ model. The models are described in Sect. 3

Panel C of Fig. 6 depicts the conditional density implied by the SV0 model. The SV0 model is capable of generating the tail-thickness. However, the plot shows that this model can reproduce only a small amount of the excess kurtosis compared to the data. The plot also shows that the conditional density is nearly symmetric. Therefore, the stochastic volatility model without the negative correlation between the stock returns and the volatility changes is unable to reproduce the negative skewness of the data.

Panel D of Fig. 6 represents the conditional density plot for the SV model. The plot indicates that the SV model provides a reasonable description of the data. The plot suggests that the SV model can generate both the excess kurtosis and the negative skewness. Compared to the reprojected conditional density for the SV0 model, the plot shows the importance of the negative return-volatility relation in capturing the negative skewness of the data. The SV model also performs better than the SV0 model in reproducing the excess kurtosis implied by the data. This result is consistent with the EMM diagnostics in Table 8. However, the density plot for the SV model suggests that the model insufficiently reproduces the excess kurtosis of the data.

As shown in Panel E of Fig. 6, the SVJ model can almost completely reproduce the shape deviations from conditional normality implied by the data. Both the excess kurtosis

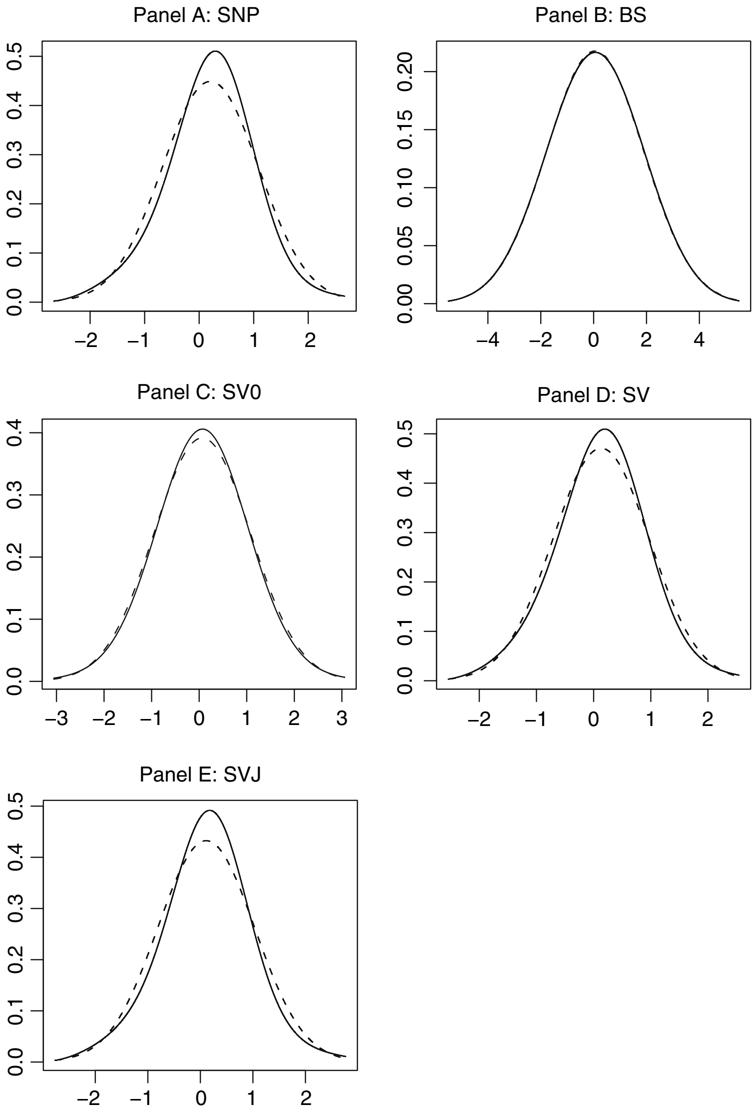

Fig. 6 Projected and reprojected conditional density: Projection, BS, SV0, SV, and SVJ. Solid lines present the projected or reprojected conditional densities. All lags are set to the unconditional mean of the data. Dashed lines present the normal densities with the same mean and variance. *Panel A* presents the conditional density implied by the SNP fit to the data (projected conditional density). *Panel B* presents the conditional density implied by the BS model, *Panel C* presents the conditional density implied by the SV0 model, *Panel D* presents the conditional density implied by the SV model, and *Panel E* presents the conditional density implied by the SVJ model. The models are described in Sect. 3

and the negative skewness reproduced by the SVJ model are quite close to those of the data. Compared to the SV model, the SVJ model shows a remarkable improvement in capturing the tail-thickness, which indicates that return jumps provide additional and significant contributions toward capturing the excess kurtosis. The model also shows a mild improvement in reproducing the negative skewness of the data. This finding is consistent with the small but significant estimate of the mean jump size in Table 7. These results indicate that return jumps are essential for capturing the conditional non-normality of the data.

5.4 Robustness check: empirical results over an extended sample period

Scott (1997) suggests that including the time series data of the S&P 500 returns in 1987 to pick up the 1987 market crash has a dramatic impact on the estimation of the SV and SJV models. As pointed by Scott (1997), the degree of negative skewness and excess kurtosis of the S&P 500 returns becomes much larger when the day of the 1987 crash is included. Interestingly, Scott (1997) finds that the SV model without return jumps can explain neither the negative skewness nor the excess kurtosis of the S&P 500 returns if the stock market crash data is included.[18]

During the period 1997–1998, which corresponds to the period of the Asian financial crisis, the KOSPI 200 shows particularly high volatility. In terms of the closing prices, the KOSPI 200 lost almost 61% of its market value from June 17, 1997 to June 16, 1998. Therefore, it is important to investigate how this volatile period affects the performance of the SV and SVJ models. This result can be compared to the results discussed in the previous subsections in which this period is not included in the analysis.

To be consistent with our estimation using the data from January 4, 2000 to July 29, 2005, the SNP and EMM estimation results in this subsection are obtained by using the daily returns of the KOSPI 200 index from January 4, 1997 to July 29, 2005 (2202 observations). Daily returns are defined as $100\cdot(\log P_t - \log P_{t-1})$, where P_t is the KOSPI 200 index price at 2:50 PM each day.

Panel A and Panel B of Fig. 1 plot the daily price and the return of the KOSPI 200 index from January 4, 1997 to July 29, 2005, respectively. As show in the plots, the volatility of the KOSPI 200 returns increases dramatically from June 1997. However, the largest market movement in a single day does not occur during the period 1997–1998. As discussed in Sect. 4, the price change limit was 12% before December 7, 1998, while it is 15% for the rest of our sample period.

Panel B of Table 5 presents the summary statistics of the KOSPI 200 returns from January 4, 1997 to July 29, 2005. There are several differences in the marginal distribution of the KOSPI 200 returns over this sample period compared to that over the period January 4, 2000 to July 29, 2005, which is presented in Panel A of Table 5. First, and as expected, when we include the data from 1997 to 1998, the standard deviation of the KOSPI 200 returns increases. The sample standard deviation is 2.0683 for the period January 4, 2000 to July 29, 2005 and it increases to 2.3462 for the period January 4, 1997 to July 29, 2005. Second, and more importantly, the marginal distribution of the longer sample period becomes closer to the normal distribution. For the period January 4, 1997 to July 29, 2005, the sample skewness is −0.0587, while it is −0.4348 for the period January 4, 2000 to July 29 2005. Similarly, the sample kurtosis computed from the daily price changes is 5.5901 for the period January 4, 1997 to July 29, 2005, while it is 6.8394 for the period January 4, 2000 to July 29 2005. Therefore, including the data from 1997 to 1998 decreases the degree of shape deviation from normality of the KOSPI 200 returns distribution. As discussed below, our SNP and EMM estimation results suggest that these changes in the distributional characteristics of the KOSPI 200 returns affect the performance of the return models.

[18] See Table 3.1 of Scott (1997).

Finally, the sample autocorrelation for the period January 4, 1997 to July 29, 2005 is 0.0936, which is statistically significant for 2202 observations. In the SNP density estimation procedure, we find that a relatively high order AR process is necessary to describe the mean dynamics of the KOSPI 200 returns data from January 4, 1997 to July 29, 2005. As suggested by ABL, such pronounced short-run return predictability is somewhat difficult to reconcile with market efficiency. It also seems to be caused by market microstructure issues such as the price change limit system in the KSE. Furthermore, as pointed out by Scott (1997), the modeling purpose of the SV and SVJ models is not for capturing the mean dynamics of equity returns. For these reasons, we prefilter the returns data using the MA(1) process and rescale the residuals to match the sample mean and variance in the original data set like ABL. This adjusted return series is then used as the raw data for the SNP and EMM estimation procedures.

Details of the SNP and EMM estimation procedures are the same as the procedures applied to the data from January 4, 2000 to July 29, 2005. Our BIC preferred SNP density for the adjusted return series from January 4, 1997 to July 29, 2005 is described by $\{L_u, L_r, L_p, K_z, \max K_z, K_x\} = \{0, 9, 1, 4, 0, 0\}$.[19] Since we use the prefiltered data, AR(0) process describes the conditional mean dynamics. There is no change in the specification of the variance dynamics compared to that of the SNP density obtained for the data from January 4, 2000 to July 29, 2005. ARCH(9) process suffices to describe the variance dynamics of the KOSPI 200 returns from January 4, 1997 to July 29, 2005. However, we need only a fourth-order Hermite polynomial in standardized innovation to capture the shape deviations from conditional normality, while a sixth-order Hermite polynomial is necessary for the data from January 4, 2000 to July 29, 2005. This change in the degree of Hermite polynomial is consistent with the decrease in the degree of negative skewness and excess kurtosis of the data.

The EMM estimation results are summarized in Table 9, which presents parameter estimates and specification tests for the return models. Table 10 presents the *t*-ratios for the scores of the best model fit with respect to the SNP auxiliary model.

As expected, the BS model is rejected at any conventional significance level. As shown by the diagnostic *t*-ratios, it fails to explain both the heteroskedastic volatility and the shape characteristic of the KOSPI 200 returns distribution. The performance of the SV0 model greatly improves the performance of the BS model. The *P*-value of the SV0 model is 0.1907. Furthermore, comparing the diagnostic *t*-ratios for the SV0 model in Tables 8 and 10 suggests that the SV0 model performs better for the period January 4, 1997 to July 29, 2005 than for the period January 4, 2000 to July 29, 2005 in capturing the characteristics of the KOSPI 200 returns distribution.

Compared to the estimation results for the period January 4, 2000 to July 29, 2005, there is an important change in the estimation results of the SV and SVJ models. As discussed in the previous subsections, the SVJ model performs much better than the SV model in terms of both the *P*-value and the *t*-ratios for the period January 4, 2000 to July 29, 2005. Return jumps provide additional flexibility in capturing the tail-behavior of the KOSPI 200 returns distribution. As presented in the last row of Table 9, however, the *P*-values of the SV and SVJ models are 0.3294 and 0.2232, respectively for the period January 4, 1997 to July 29, 2005. The value of the EMM objective function for the SVJ model is 9.4304, which is smaller than that for the SV model, which is 11.3710. However, the deterioration in the *P*-value indicates that this mild improvement in the objective function is insufficient to be compensated by the degrees-of-freedom loss. Therefore, the SV model is the best in

[19] The results are not reported here but are available upon request.

Table 9 Parameter estimates and specification tests for the period January 4, 1997 to July 29, 2005

Parameter	BS	SV0	SV	SVJ
μ	0.3589 (0.0031)	0.3028 (0.0046)	0.1963 (0.0046)	0.1909 (0.0030)
σ	0.2028 (0.0009)			
κ		0.7924 (0.0094)	0.9358 (0.0108)	0.8416 (0.0081)
θ		0.0744 (0.0010)	0.0893 (0.0008)	0.0878 (0.0007)
σ_V		0.2252 (0.0018)	0.3041 (0.0020)	0.2930 (0.0013)
ρ			−0.5948 (0.0208)	−0.5605 (0.0198)
λ				2.9719 (0.3074)
μ_J				−0.0254 (0.0013)
δ				0.0053 (0.0022)
χ^2	41.4130	14.8250	11.3710	9.4304
df	13	11	10	7
P-value	0.0001	0.1907	0.3294	0.2232

This table presents parameter estimates and goodness-of-fit tests for the stock return models. The model and parameters are described in Sect. 3. The parameter estimates are expressed in decimal form on an annual basis. Standard errors are given in parentheses. The last three rows report χ^2 statistics for the goodness-of-fit of the models, degrees of freedom, and corresponding *P*-value

Table 10 Diagnostic *t*-ratios for the period January 4, 1997 to July 29, 2005

		BS	SV0	SV	SVJ
AR	b_0	−0.1442	0.1884	−0.0227	−0.1174
ARCH	R_0	−2.7425	−0.5898	−0.3669	−0.2145
ARCH	P_1	−1.4919	0.6360	0.8309	0.6848
ARCH	P_2	−3.2495	−0.9996	−0.6232	−0.6346
ARCH	P_3	2.9665	−0.6942	−0.9796	−0.7915
ARCH	P_4	−4.0103	−1.3889	−1.1266	−1.0354
ARCH	P_5	−3.8530	−1.3471	−1.1641	−1.1980
ARCH	P_6	3.4771	1.1756	1.0273	0.8177
ARCH	P_7	−2.1632	1.0576	1.3750	1.1914
ARCH	P_8	−3.4091	−0.9142	−0.7889	−0.7003
ARCH	P_9	4.0135	0.9025	0.5450	0.4958
Hermite	a_{01}	0.2008	1.1848	0.1081	0.0780
Hermite	a_{02}	−5.3317	−1.0188	−0.7985	−0.6773
Hermite	a_{03}	−0.1439	0.6476	−0.1165	−0.5452
Hermite	a_{04}	−0.4136	−0.1511	−0.0813	0.5160

This table presents *t*-ratio diagnostics for EMM scores evaluated for the SNP score generator 091400. The *t*-ratios are the test statistics of the null hypothesis that the scores with respect to the parameters of the SNP density are equal to zero

explaining the KOSPI 200 returns distribution for this period. This finding indicates that compared to the importance of stochastic volatility, the additional role of return jumps in capturing the dynamics of the KOSPI 200 returns is not evident before year 2000.

Comparing the diagnostic *t*-ratios for the SV and SVJ models also delivers a similar picture. All the *t*-ratios for the SV model presented in Table 10 are below 2.0 and are fairly small. The *t*-ratios for the SVJ model suggest that there is no remarkable improvement in capturing the volatility dynamics or the shape characteristic of the data compared to those for the SV model.

Although all of the jump parameters are statistically significant, both the jump frequency and the range of the typical jumps are dramatically decreased compared to those for the period January 4, 2000 to July 29, 2005. As shown in Tables 7 and 9, the estimated value of the jump frequency, λ, decreases from 7.0940 to 2.9719. The estimated value of the mean jump size, μ_J, increases from -0.32% to -2.54%. However, the variability of jump size, δ, decreases 2.58–0.53%.

In summary, the EMM estimation results based on the two data sets deliver important implications about the ingredients of the KOSPI 200 returns generating mechanism. First, the estimation results for the two data sets commonly indicate that both the SV and SVJ models perform quite well in capturing the dynamics of the KOSPI 200 returns. Second, the stochastic volatility factor plays an important role in explaining the KOSPI 200 returns distribution for the entire sample period. However, the SV model cannot fully capture the tail-behavior of the KOSPI 200 returns distribution for the period January 4, 2000 to July 29, 2005. Finally, the role of return jumps becomes evident after year 2000. Especially, the greatest contribution of discrete jump factor lies in its ability to fit the tail-behavior of the KOSPI 200 returns distribution. The SVJ model almost completely captures the dynamics of the KOSPI 200 returns for the period January 4, 2000 to July 29, 2005 by successfully complementing the insufficient performance of the SV model.

6 Option pricing applications

In this section, we investigate the option pricing implications of the EMM estimation results for the SV and SVJ models based on the period January 4, 2000 to July 29, 2005. Time-series results indicate that both the diffusive stochastic volatility and the discrete return jump factors are important ingredients in the KOSPI 200 returns generating mechanism. Although the role of return jumps is statistically significant, the SV model that incorporates only the stochastic volatility is not misspecified for explaining the KOSPI 200 returns dynamics. Therefore, it is of interest to compare the performance of the SV and SVJ models in capturing the cross-sectional behavior of option prices.

The purpose of this section is to compare the option pricing performance of the SV and SVJ models by exploiting the information in the cross-section of the KOSPI 200 options prices. First, we compare the option pricing errors across the moneyness and time-to-maturity for an in-sample period by using the daily prices of the KOSPI 200 options from August 1, 2003 to July 29, 2005. For this purpose, we estimate the risk premiums of volatility and jump risks together with the daily spot volatility. The parameters not affected by the risk adjustment are fixed at the EMM estimates in Table 7.[20] We also compare the shape of the BS implied volatility (IV) curves implied by the models with those implied by the observed option prices. Finally, we examine the models' performance in the out-of-sample option pricing by using an extended sample period that is not used for the

[20] Therefore, the overall estimation procedure in this paper is a two-step scheme wherein the structural parameters of return dynamics are estimated from the time-series of stock returns, and then the volatility and jump risk premia are estimated from the cross-section of option prices. Our approach is similar to those of Fiorentini et al. (2002), Jiang (2002), and Jiang and van der Sluis (1999.

estimation of the model parameters under the physical probability measure. To compute the theoretical option price, we use the closed-form option pricing solution of Bates (1996) and Scott (1997) derived from the Fourier transform method.[21]

6.1 Options data and methodology

6.1.1 Options data

The KOSPI 200 options market is characterized as a market of the massive trading activities and of the active participation of individual investors. Since it was introduced in July 7 1997, the KOSPI 200 options are the most actively traded derivatives among all of the world's exchange-traded derivatives from 2000 to 2005 as noted in Sect. 1. Its daily trading volume is 8.2 million contracts in December 2005. Another characteristic of this market is the preponderance of individual investors. As shown in Table 11, individual investors account for 42.85% of total trading volume in year 2005.[22] As pointed out by Upper (2005), this degree of participation of individual investors is far greater than in other countries' options markets.

Three consecutive near-term delivery months and one additional month from the quarterly cycle (March, June, September, and December) make up four contract months. Trading in the KOSPI 200 index option is fully automated. The exercise style of the KOSPI 200 index options is European.

The sample period extends from August 1, 2003 to July 31, 2006. Option prices from August 1, 2003 to July 29, 2005 are used for the comparison of the in-sample pricing errors and the BS IV curves. Option prices from August 1, 2005 to July 31, 2006 are used for the investigation of the out-of-sample pricing performance. Minute-by-minute transaction prices for the KOSPI 200 options are obtained from the KSE. The 91-day certificate of

Table 11 Trading volume by investor group at the KOSPI 200 options market

	2000	2001	2002	2003	2004	2005
Domestic individual investors	275 (70.84)	1,190 (72.24)	2,488 (65.83)	3,110 (54.79)	2,518 (49.93)	2,172 (42.85)
Domestic securities companies	78 (22.09)	287 (19.70)	928 (25.80)	1,801 (32.90)	1,805 (37.32)	2,017 (42.01)
Foreign investors	20 (5.27)	103 (6.27)	280 (7.40)	627 (11.06)	602 (11.93)	730 (14.39)
Other domestic institutions	7 (1.79)	29 (1.78)	37 (0.97)	71 (1.26)	41 (0.82)	38 (0.76)

This table presents the trading volume by four investor groups at the KOSPI 200 options market from 2000 to 2005. The trading volume is the total number of short and long contracts traded in each year and in million of contracts. The percentages of total trading volume by four investor groups are given in parentheses. Data are obtained from the Korea Exchange (KRX, http://www.krx.co.kr)

[21] In calculating the theoretical option price, a subroutine QAGI by Piessens et al. (1983) of ?IMSL C library is applied to numerically integrate the imaginary part of the complex Fourier transform.

[22] As shown in the third row of Table 11, the participation of foreign investors in the KOSPI 200 options market is on a steady increase. It was 5.27% at the end of 2000 and has increased to 14.39% at the end of 2005.

deposit (CD) yields are used as risk-free interest rates.[23] Since the KOSPI 200 options are European-style, index levels are adjusted for dividend payments before each option's expiration date. The KOSPI 200 index pays dividends only at the end of March, June, September and December, which are used for adjustment dates.

The following rules are applied to filter the options data needed for the empirical test. First, to ease computational burden, for each day in the sample, only the last reported transaction price prior to 2:50 PM of each option contract is employed in the empirical test.[24] That is, an option of a particular moneyness and maturity is represented only once in the sample. The KOSPI 200 index price is simultaneously observed as the option's transaction price, which avoids the issue of non-synchronous prices. Third, as options with less than 6 days or more than 90 days to expiration may induce liquidity-related biases, they are excluded from the sample. Fourth, to mitigate the impact of price discreteness on option valuation, prices lower than 0.02 are not included. Fifth, only out-of-the-money options are included in the sample.[25] That is, we use only call option prices when the moneyness is less than one. Similarly, we use only put option prices when the moneyness is larger than one. Moneyness is defined as the ratio of KOSPI 200 price to strike price. The trading volumes of OTM options are larger than those of other moneyness options. Finally, prices not satisfying the arbitrage restriction are excluded. Average number of daily call (put) options used in the estimation is 15.23 (19.59) for the period August 1, 2003 to July 29, 2005 and 19.64 (25.04) for the period August 1, 2005 to July 31, 2006.

6.1.2 Methodology

We estimate the risk premiums of volatility and jump risks together with the spot volatility. For the investigation of both the in-sample and out-of-sample pricing performance, we compare the BS model that has the volatility as the only parameter, with the SV and SVJ models. To compare the pricing errors of the BS model with those of other models, we suppose that the spot volatility is a free parameter for all models. This assumption is consistent with Bakshi et al. (1997), Bates (1996), Duffie et al. (2000), Jiang (2002), and Kim and Kim (2004, 2005).

Following Bates (1996) and Scott (1997), we assume that the dynamics of the stock price and the return variance under the risk neutral probability measure are

$$\frac{\mathrm{d}S(t)}{S(t)} = \left(r - \lambda^* \mu_J^*\right)\mathrm{d}t + \sqrt{V(t)}\mathrm{d}w_S^*(t) + J^*(t)\mathrm{d}q^*(t), \tag{10}$$

$$\mathrm{d}V(t) = [\kappa\theta - (\kappa + \xi)V(t)]\mathrm{d}t + \sigma_V\sqrt{V(t)}\mathrm{d}w_V^*(t), \tag{11}$$

$$\mathrm{cov}(\mathrm{d}w_S^*(t), \mathrm{d}w_V^*(t)) = \rho\mathrm{d}t, \tag{12}$$

$$\mathrm{prob}(\mathrm{d}q^*(t) = 1) = \lambda^*\mathrm{d}t, \tag{13}$$

$$\ln(1 + J^*(t)) \sim N(\ln(1 + \mu_J^*) - \frac{1}{2}\delta^2, \delta^2), \tag{14}$$

[23] Korea does not have a liquid Treasury bill market, therefore the 91-day certificate of deposit (CD) yields are used in spite of the mismatch of maturity of options and interest rates.

[24] As discussed in Sect. 4, there are simultaneous bids and offers from 2:50 PM to 3:00 PM in the KSE.

[25] We thank an anonymous referee for recommending us this approach.

where r is the instantaneous riskless interest rate, w_S^* and w_V^* are the standard Brownian motions under the risk neutral probability measure, ξ is the market price of volatility risk, q^* is a Poisson process with intensity λ^*, and $J^*(t)$ is the jump size under the risk neutral probability measure.

We estimate ξ and $V(t)$ for the SV model and ξ, λ^*, μ_J^*, and $V(t)$ for the SVJ model by using the actual option prices. The parameter vector $\varphi = \{\mu, \kappa, \theta, \sigma_V, \rho, \delta\}$ which are not affected by the risk adjustment are fixed to the EMM estimates $\hat{\phi}$ in Table 7. Since the closed-form solutions are available for an option price, a natural candidate for the estimation of parameters which enter the pricing formula is a non-linear least squares procedure involving minimization of the sum of squared errors between the model and market prices. As estimated in the standard practice, we estimate the parameters of each model every sample day.

Let $O_i\ (t,\tau\ ;K)$ denote the observed price of option i on day t for a given strike price K and τ days to maturity and let $O_i^*(t,\tau;\hat{\phi},\phi^*,V(t),K)$ denote the model price for given parameter vectors $\hat{\phi}$ and φ^* with a spot volatility $V(t)$. To estimate the risk-neutral parameters of each model, we minimize the sum of squared percentage pricing errors between the model and the market prices:

$$\min_{\phi^*,V(t)} \sum_{i=1}^{N} \left[\frac{O_i^*(t,\tau;\hat{\phi},\phi^*,V(t),K) - O_i(t,\tau;K)}{O_i(t,\tau;K)}\right]^2, \quad (t = 1,\ldots,T),$$

where N denotes the number of options on day t, φ^* denotes the parameter vector to be estimated, i.e., $\{\xi\ ,\ \lambda^*,\ \mu_J^*\}$, and T denotes the number of days in the sample. Conventionally, the objective function to minimize the sum of squared errors is used. However, we adopt the above function since the conventional method that gives more weight to relatively expensive options (e.g., long term options) makes a bad fit for relatively cheap options (e.g., short-term options).

6.2 Pricing performance for the period August 1, 2003 to July 29, 2005

Table 13 presents the mean absolute percentage pricing errors (MAPEs) of different model specifications across the moneyness and time-to-maturity for the in-sample period August 1, 2003 to July 29, 2005.[26] For better exposition, we employ six fixed partitions for the degree of moneyness and three independent partitions for the time-to-maturity, which results in 18 categories. For each category, the MAPEs are obtained by averaging the absolute percentage pricing errors of all options in that category. It should be noted that the MAPEs of the options with the moneyness larger than one are computed from put options and the MAPEs of the options with the moneyness less than one are computed from call options.

In order to assess the magnitude of the pricing errors, we use the BS pricing model as a benchmark. First, the SVJ model outperforms both the BS and SV models for all moneyness and maturity categories. As shown in the last row of Table 13, the average MAPE becomes smaller as we move from the BS model to the SVJ model. (BS: 42.39%, SV: 22.34%, SVJ: 10.74%) Second, the pricing errors of short-term ($\tau < 30$) options are larger than those of mid-term ($30 \leq \tau < 60$) and long-term ($60 \leq \tau < 90$) options for all models.

[26] For each model, Table 12 presents the average and standard errors (in parentheses) of the risk-neutral parameters, which are estimated daily.

For the SVJ model, however, the differences of pricing errors across the time-to-maturity are small. Especially, the superiority of the SVJ model over the SV model is most remarkable in short-term options. That is, the difference of pricing errors between the SV and SVJ models decreases as the maturity increases. Our finding suggests that the importance of return jumps in fitting the option prices increases as the maturity decreases. Third, across the moneyness, the pricing errors of both OTM call options ($S/K < 0.97$) and OTM put options ($S/K \geq 1.03$) are larger than those of near-the-money options ($0.97 \leq S/K < 1.03$) for all models. For the BS and SV models, this phenomenon tends to be more evident for short-term options. For the SVJ model, these differences of pricing errors among the moneyness are smaller than those of the BS and SV models for all maturities. Therefore, the SVJ model performs better than the BS and SV models in explaining the option prices across the moneyness of options.

Our comparison of the in-sample pricing errors suggests that return jumps play an important role in improving the option pricing performance for short-term options. For more investigation of this superiority of the SVJ model over the SV model, Fig. 7 depicts the BS implied volatilities extracted from the observed option prices and the theoretical option prices obtained by the SV and SVJ models for short-term options. The BS IV values are obtained as follows. First, we extract the BS IV values for the observed option prices by using the BS option pricing formula. Second, the BS IV values for the SV and SVJ models are obtained by substituting the estimated option prices of each model for the observed option prices in the BS option pricing formula. Finally, the IV values in each of the moneyness partition are averaged for a given period.

The plots indicate that the existence of jump risk premia is important for the structural fitting of the systematic variations in the BS implied volatilities. In particular, the SVJ model performs much better than the SV model in capturing both the level and the shape of the IV curves. The SV model under-prices the OTM put options and over-prices the OTM call options. More importantly, the SV model has difficulty in fitting the steepness of the volatility smirks. Especially, the SV model fails to fit the BS IV values of OTM put options. As shown in the plots, the SVJ model performs well in capturing both the level and the shape of the BS IV curves. Our result suggests that return jumps are crucial in capturing the volatility smirks of the short-term KOSPI 200 options as well as in explaining the KOSPI 200 returns distribution.

6.3 Out-of-sample pricing performance

In-sample pricing performance can be biased due to the potential problem of over-fitting the data. A good in-sample fit might be a consequence of having an increasingly larger number of structural parameters. To lower the impact of this connection to inferences, we turn to examine the out-of-sample pricing performance. In the out-of-sample pricing, the presence of more parameters may actually cause over-fitting and have the model penalized if the extra parameters do not improve its structural fitting. For this purpose, we exploit the KOSPI 200 options prices from August 1, 2005 to July 31, 2006, which extends 1 year from the EMM estimation period, i.e., January 4, 2000 to July 29, 2005.[27]

[27] Therefore, out-of-sample approach used in this paper is different from that of Bakshi et al. (1997), in which the pricing performance evaluations rely on in-sample one-step ahead forecasts.

Table 12 Estimates of risk-neutral parameters

Parameter	BS	SV	SVJ
Panel A: August 1, 2003 to July 29, 2005			
ξ		0.9022 (0.0674)	1.3507 (0.0880)
λ^*			0.3481 (0.1062)
μ_J^*			−0.1039 (0.0572)
$V(t)$	0.0520 (0.0010)	0.0551 (0.0012)	0.0511 (0.0012)
Panel B: August 1, 2005 to July 31, 2006			
ξ		1.1697 (0.0957)	1.8831 (0.1109)
λ^*			0.4426 (0.0406)
μ_J^*			−0.0856 (0.0785)
$V(t)$	0.0443 (0.0007)	0.0476 (0.0010)	0.0403 (0.0010)

This table presents the risk-neutral parameters of a given model, which are estimated by minimizing the sum of squared percentage pricing errors between the market price and the model price for each option. The daily average of the estimated parameters is presented first, followed by its standard error in parentheses. The parameter estimates are expressed in decimal form on an annual basis. Return dynamics under the risk-neutral probability measure are described in Eqs. (10) to (14)

First, we examine the usefulness of the parameters estimated under the physical probability measure, which is necessary to verify our investigation of the out-of-sample pricing performance. If there is a structural break in the parameters that govern the return dynamics, then the models' performance in the out-of-sample period can be penalized even though there is no serious over-parameterization. We estimate the risk premiums of volatility and jump risks together with the spot volatility by using the option prices from August 1, 2005 to July 31, 2006. In the same manner as the previous subsection, model parameters not affected by the risk adjustment are fixed at the EMM estimates in Table 7. This practice does not come under the category of in-sample pricing. But we call this practice the in-sample pricing because we price options by using the risk-neutral parameters and spot volatility which are estimated from the current option prices.

Table 14 presents these in-sample pricing errors of each model. For all models, the overall levels of the MAPEs are slightly larger compared to those presented in Table 13. However, the qualitative features of pricing errors are quite similar to those in Table 13. The SVJ model shows the best performance in spite of using the past structural parameters that describe return dynamics under the physical probability measure.

Given the validity of our out-of-sample investigation, Table 15 presents 1-day ahead out-of-sample pricing errors. We use the current day's estimated structural parameters and spot volatility to price options on the following day.

As expected, all three models experience deterioration in fitting the option prices. As in the in-sample pricing results presented in Tables 13 and 14, the SVJ model performs better than both the BS and SV models. (BS: 45.90%, SV: 27.14%, SVJ: 18.91%) The pricing errors decrease as the maturity increases for all models. However, the superiority of the SVJ model over the SV model for short-term options is evident. Similarly for the in-sample results, all models show moneyness-based pricing errors. The SVJ model shows smaller moneyness-based pricing errors than other models for all maturities. However, its superiority over the SV model in fitting the option prices across the moneyness decreases.

Table 13 In-sample option pricing errors for the period August 1, 2003 to July 29, 2005

Maturity	Moneyness	BS	SV	SVJ
$T < 30$	< 0.94	0.5758	0.2505	0.1932
	0.94–0.97	0.3877	0.2178	0.1142
	0.97–1.00	0.1727	0.1290	0.0852
	1.00–1.03	0.1357	0.1261	0.1080
	1.03–1.06	0.3010	0.1965	0.1313
	≥ 1.06	0.7447	0.5111	0.1430
	Subtotal	0.4619	0.2909	0.1341
$30 \leq T < 60$	< 0.94	0.6359	0.2196	0.1215
	0.94–0.97	0.2612	0.1659	0.0888
	0.97–1.00	0.1183	0.1010	0.0780
	1.00–1.03	0.0993	0.0982	0.0746
	1.03–1.06	0.1873	0.1247	0.0836
	≥ 1.06	0.6193	0.3126	0.1063
	Subtotal	0.4375	0.2153	0.0992
$60 \leq T < 90$	< 0.94	0.4999	0.1589	0.1079
	0.94–0.97	0.2003	0.1457	0.1047
	0.97–1.00	0.1155	0.1075	0.0914
	1.00–1.03	0.1112	0.0985	0.0634
	1.03–1.06	0.1784	0.1239	0.0696
	≥ 1.06	0.4846	0.2145	0.0909
	Subtotal	0.3636	0.1664	0.0927
All	< 0.94	0.5786	0.2081	0.1347
	0.94–0.97	0.2838	0.1764	0.1014
	0.97–1.00	0.1360	0.1121	0.0838
	1.00–1.03	0.1151	0.1088	0.0852
	1.03–1.06	0.2215	0.1476	0.0952
	≥ 1.06	0.6112	0.3337	0.1110
	Subtotal	0.4239	0.2234	0.1074

This table presents in-sample MAPEs (mean absolute percentage pricing errors) with respect to moneyness and time-to-maturity. S/K is defined as moneyness, where S denotes the KOSPI 200 index price and K denotes the strike price. Each model is estimated every day during the sample period. The MAPEs are expressed in decimal form

To summarize, although the differences among the models decrease in the case of the out-of-sample results compared to those of the in-sample pricing results, out-of-sample pricing confirms the superiority of the SVJ model.

7 Conclusion

This paper investigates the performance of the stochastic volatility and the stochastic volatility with return jumps models in explaining the conditional distribution of the KOSPI 200 returns. Model parameters are estimated by the EMM procedure that recently has been extended by employing the MCMC method as its computational strategy.

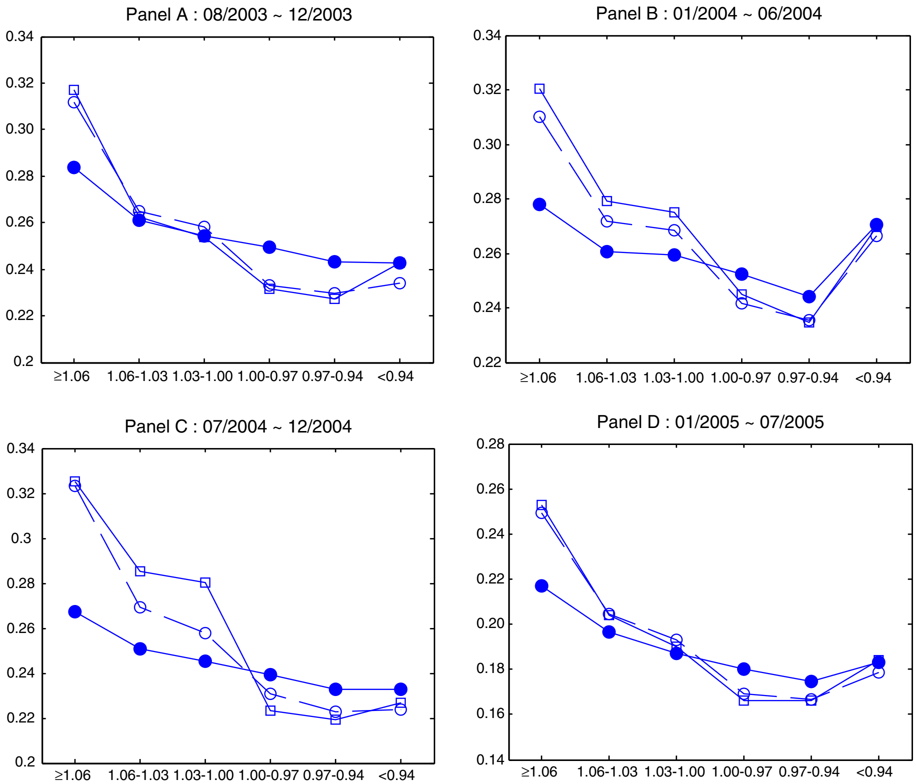

Fig. 7 Black-Scholes implied volatilities for short-term options. This figure presents the BS IV values implied by the observed option prices and the estimated option prices obtained by the SV and SVJ models for short-term ($\tau < 30$) options. The X-axis denotes the moneyness of options, S/K, where S denotes the KOSPI 200 index price and K denotes the strike price. The Y-axis denotes the BS IV values. The IV values of the options with the moneyness larger than 1.0 are computed from put options and the IV values of the options with the moneyness less than 1.0 are computed from call options. In each panel, solid line represents the BS IV curve extracted from the observed option prices, dotted line represents the BS IV curve extracted from the estimated option prices obtained by the SV model, and dashed line represents the BS IV curve extracted from the estimated option prices obtained by the SVJ model

Using the semiparametric ARCH model as a consistent estimator of the conditional density of the KOSPI 200 returns, we find that both stochastic volatility and return jumps are important ingredients of the KOSPI 200 return dynamics. Both the SV and SVJ models cannot be rejected at any conventional significance level.

In the SV model, the negative value of the return-volatility correlation plays an important role in capturing the conditional negative skewness. However, the SV model insufficiently reproduces the conditional excess kurtosis of the data for the period January 4, 2000 to July 29, 2005. The SVJ model almost completely reproduces the conditional volatility, the negative skewness, and the tail-thickness implied by the KOSPI 200 returns for the period January 4, 2000 to July 29, 2005. We find that jump factor is essential for capturing the conditional excess kurtosis of the data for this period. However, this role of return jumps is not evident before year 2000. Our extended sample period analysis indicates that the SV model performs better than the SVJ model

Table 14 In-sample option pricing errors for the period August 1, 2005 to July 31, 2006

Maturity	Moneyness	BS	SV	SVJ
$\tau < 30$	< 0.94	0.6621	0.2643	0.1641
	0.94–0.97	0.4535	0.2549	0.1204
	0.97–1.00	0.1661	0.1301	0.0797
	1.00–1.03	0.1608	0.1377	0.1193
	1.03–1.06	0.3519	0.2389	0.1507
	$\geq$ 1.06	0.7947	0.5497	0.1478
	Subtotal	0.4951	0.3161	0.1318
$30 \leq \tau < 60$	< 0.94	0.7011	0.2036	0.1290
	0.94–0.97	0.3396	0.2416	0.1264
	0.97–1.00	0.1534	0.1433	0.1037
	1.00–1.03	0.1361	0.1353	0.1268
	1.03–1.06	0.2456	0.1769	0.1415
	$\geq$ 1.06	0.6527	0.3401	0.1567
	Subtotal	0.4525	0.2359	0.1358
$60 \leq \tau < 90$	< 0.94	0.7508	0.1327	0.1020
	0.94–0.97	0.2933	0.2197	0.1508
	0.97–1.00	0.1716	0.1676	0.1359
	1.00–1.03	0.1467	0.1535	0.1436
	1.03–1.06	0.2359	0.1871	0.1646
	$\geq$ 1.06	0.5500	0.2451	0.1915
	Subtotal	0.4195	0.1931	0.1527
All	< 0.94	0.7078	0.1952	0.1286
	0.94–0.97	0.3599	0.2387	0.1323
	0.97–1.00	0.1626	0.1456	0.1046
	1.00–1.03	0.1474	0.1403	0.1280
	1.03–1.06	0.2767	0.1998	0.1513
	$\geq$ 1.06	0.6704	0.3807	0.1629
	Subtotal	0.4564	0.2487	0.1393

This table presents in-sample MAPEs (mean absolute percentage pricing errors) with respect to moneyness and time-to-maturity. S/K is defined as moneyness, where S denotes the KOSPI 200 index price and K denotes the strike price. Each model is estimated every day during the sample period. The MAPEs are expressed in decimal form

for the period January 4, 1997 to July 29, 2005 in terms of the P-value of the EMM specification test. Therefore, our results suggest that the importance of return jumps in explaining the KOSPI 200 returns distribution has substantially increased in recent years.

Our investigation of the option pricing performance suggests that the SVJ model performs better than the BS and SV models in pricing the KOSPI 200 options in terms of both the in-sample and out-of-sample pricing errors. The superiority of the SVJ model is particularly remarkable for short-term options. We also find that return jumps are essential in capturing the systematic variations in the BS IV curves, i.e., the volatility smirks observed in short-term options.

Table 15 1-day ahead out-of-sample pricing errors for the period August 1, 2005 to July 31, 2006

Maturity	Moneyness	BS	SV	SVJ
$\tau < 30$	< 0.94	0.7262	0.4109	0.3762
	0.94–0.97	0.4814	0.3388	0.2391
	0.97–1.00	0.1746	0.1495	0.1120
	1.00–1.03	0.1671	0.1594	0.1458
	1.03–1.06	0.3497	0.2592	0.1878
	$\geq$ 1.06	0.7865	0.5499	0.2567
	Subtotal	0.5062	0.3540	0.2233
$30 \leq \tau < 60$	< 0.94	0.7013	0.2558	0.1978
	0.94–0.97	0.3382	0.2496	0.1492
	0.97–1.00	0.1559	0.1449	0.1122
	1.00–1.03	0.1441	0.1395	0.1375
	1.03–1.06	0.2465	0.1841	0.1597
	$\geq$ 1.06	0.6498	0.3539	0.2127
	Subtotal	0.4527	0.2523	0.1743
$60 \leq \tau < 90$	< 0.94	0.7257	0.1849	0.1558
	0.94–0.97	0.2996	0.2277	0.1649
	0.97–1.00	0.1778	0.1685	0.1435
	1.00–1.03	0.1536	0.1568	0.1509
	1.03–1.06	0.2379	0.1888	0.1737
	$\geq$ 1.06	0.5446	0.2571	0.2092
	Subtotal	0.4161	0.2085	0.1732
All	< 0.94	0.7150	0.2697	0.2264
	0.94–0.97	0.3700	0.2701	0.1820
	0.97–1.00	0.1682	0.1529	0.1207
	1.00–1.03	0.1545	0.1506	0.1435
	1.03–1.06	0.2770	0.2096	0.1729
	$\geq$ 1.06	0.6652	0.3899	0.2255
	Subtotal	0.4590	0.2714	0.1891

This table presents 1-day ahead out-of-sample MAPEs (mean absolute percentage pricing errors) with respect to moneyness and time-to-maturity. S/K is defined as moneyness, where S denotes the KOSPI 200 index price and K denotes the strike price. Each model is estimated every day during the sample period. One day ahead out-of-sample pricing errors are computed with estimated parameters from current day. The MAPEs are expressed in decimal form

Therefore, we conclude that return jumps play an important role in explaining both the time-series behavior of the KOSPI 200 returns and the cross-sectional behavior of the KOSPI 200 options prices.

Appendix A

Implementation of EMM

The EMM procedure can be thought of as a two-step process. The first step is finding the score generator. Gallant and Long (1997) show that if the score generator is a seminonparametric (SNP) density, then the efficiency of the EMM estimator can be made close to that of maximum likelihood. In a similar vein, Gallant and Tauchen (2002) recommend the SNP density as a general purpose score generator and that is our choice in this paper. We employ the latest specification of the SNP score generator presented by Gallant and Tauchen (2005). We use a common SNP density to estimate the parameters of the four stock return models, which allows us to rank all the models according to the *P*-values implied by the EMM criterion function. Details of the SNP density are presented in the Appendix B. The second step of the EMM procedure involves estimating the parameter vector for the stock return model. This step requires simulating long stock return series. We simulate 50,000 daily *log* returns by a Euler discretization scheme. To estimate the annualized model parameters, we assume that there are 245 trading days per year and one trading day is set equal to 1/245 of a year. To reduce the discretization bias, one trading day is divided into 24 subintervals. Therefore, the discretization interval is 1/(24·245). The daily data set is generated by retaining every 24th simulated value. There is a burn in period of 5,000 to eliminate simulation transients. We follow the approach of CGGT to simulate return jumps. Intervals between return jumps are randomly generated from an exponential distribution. The mean of the exponential distribution is the reciprocal of the jump intensity. If the intervals fall inside the discretization interval, then the jump size is randomly generated from the normal distribution whose mean and variance are defined in Eq. (5). Therefore, the jump intensity parameter has annual scaling.

Finally, Gallant and Tauchen (2005) adopt Bayesian Markov Chain Monte Carlo (MCMC) methods to evaluate the EMM criterion function, while the previous EMM procedure uses a derivative based hill climbing method. As noted previously, this extension is important for the purpose of this paper because the MCMC algorithms are suited for the estimation of the SVJ model. Chernozhukov and Hong (2003) provide theoretical underpinnings for exploiting the Bayesian MCMC methods as the computational strategy in the EMM estimation procedure. In short, the computational strategy involves applying the Bayesian MCMC methods with $L(\mathrm{P}) = -n\, s_n(\mathrm{P})$ as the likelihood where $s_n(\mathrm{P})$ is the EMM criterion function. See Gallant and Tauchen (2005) for details of the extended EMM procedure.

In this paper, we use a uniform prior to reflect the price change limit in the KSE. The prior evaluates the maximum of the absolute values of the simulated returns data generated by a candidate parameter vector. If the maximum exceeds 15%, then the value of the parameter vector is rejected in the MCMC procedure. The Metropolis-Hastings algorithm (Hastings 1970; Metropolis et al. 1953) is employed to sample from the target density $L(\mathrm{P})$. We use a random walk, single move, and normal proposal density. If the MCMC chain shows strictly linear relationships for some group of the model parameters, then a group move scheme is implemented by employing a multivariate normal proposal density.

Appendix B

SNP density

Since first introduced by Gallant and Tauchen (1989), the SNP method has experienced several important extensions. In this paper, we adopt the latest specification of the SNP density as the score generator, which was developed by Gallant and Tauchen (2005). They employ the BEKK-GARCH of Engle and Kroner (1995) to model the second moment dynamics.

In the SNP procedure, a conditional density for a stationary time series is approximated by a Hermite polynomial expansion. Let f_K (y_t | x_{t-1} , Θ) denote the SNP density where y_t denotes the current observation, x_{t-1} denotes the lagged observations, and Θ denotes the K-dimensional parameter vector of the SNP density. In our application, y_t is a series of daily stock returns. The form of the SNP density is a Hermite polynomial in standardized innovation times a Gaussian autoregression (AR) with the potential for ARCH innovations. Denoting a demeaned transformation of the stock returns process y_t as $z_t = R^{-1}_{x_{t-1}}(y_t - \mu_{x_{t-1}})$ where the conditional mean function $\mu_{x_{t-1}}$ is an AR on L_u lags

$$\mu_{x_{t-1}} = b_0 + \sum_{j=1}^{Lu} B_j y_{t-j}, \tag{B1}$$

and the conditional variance function $\sum_{x_{t-1}} = R^2_{x_{t-1}}$ is a BEKK-type ARCH on L_r lags

$$\sum_{x_{t-1}} = R_0^2 + \sum_{j=1}^{Lr} P_i^2 \left(y_{t-j} - \mu_{x_{t-1-j}}\right), \tag{B2}$$

the SNP density of z_t is given by

$$f_K(z_t|x_{t-1}, \Theta) = \frac{[P(z_t, x_{t-1})]^2 \phi(z_t)}{\int [P(u, x_{t-1})]^2 \phi(u) \mathrm{d}u}, \tag{B3}$$

$$P(z_t, x_{t-1}) = \sum\nolimits_{|\alpha|=0}^{K_z} \left(\sum\nolimits_{|\beta|=0}^{K_x} a_{\beta\alpha} x_{t-1}^{\beta} \right) z_t^{\alpha}, \tag{B4}$$

where P (z_t , x_{t-1}) is a polynomial in (z_t , x_{t-1}) of degree (K_z , K_x) and φ (z_t) denotes the standard normal density function.

Departures from normality are captured via the polynomial. If K_z = 0, then the SNP density is Gaussian. On the contrary, if K_z > 0, then the SNP method can accurately approximate densities from a large class that includes densities with fat, t-like tails, densities with tails that are thinner Gaussian, and skewed densities (Gallant and Nychka 1987; Gallant and Tauchen 2002). If K_x = 0, then the SNP density f_K (z_t | x_{t-1}, Θ) does not depend on x_{t-1} and therefore is homogeneous. On the contrary, if K_x > 0, then the shape of the SNP density depends on the lags of the process y_t, and therefore is heterogeneous. The number of lags in the x_{t-1} part of the polynomial is L_p. With a large K_z, the number parameters can be too large to allow for conditional heterogeneity by the coefficients of the polynomials. Gallant and Tauchen (2005) control this by restricting the dependence on x_{t-1} of the polynomial coefficients of a degree higher than max K_z to be zero. That is, max K_z denotes the maximum degree of K_z that depends on x_{t-1}. The tuning parameters of an SNP density are, therefore, {L_u, L_r, L_p, K_z, max K_z, K_x}.

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