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# Empirical comparison of alternative stochastic volatility option pricing models: Evidence from Korean KOSPI 200 index options market

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**Abstract**

This article investigates the improvement in the pricing of Korean KOSPI 200 index options when stochastic volatility is taken into account. We compare empirical performances of four classes of stochastic volatility option pricing models: (1) the ad hoc Black and Scholes procedure that fits the implied volatility surface, (2) Heston and Nandi's [Rev. Financ. Stud. 13 (2000) 585] GARCH type model, (3) Madan et al.'s [Eur. Financ. Rev. 2 (1998) 79] variance gamma model, and (4) Heston's [Rev. Financ. Stud. 6 (1993) 327] continuous-time stochastic volatility model. We find that Heston's model outperforms the other models in terms of effectiveness for in-sample pricing, out-of-sample pricing and hedging. Looking at valuation errors by moneyness, pricing and hedging errors are highest for out-of-the-money options, and decrease as we move to in-the-money options in all models. The stochastic volatility models cannot mitigate the "volatility smiles" effects found in cross-sectional options data, but can reduce the effects better than the Black and Scholes model. Heston and Nandi's model shows the worst performance, but the performance of the Black and Scholes model is not too far behind the stochastic volatility option pricing model.


*JEL classification:* G13
*Keywords:* Option pricing model; Out-of-sample pricing; Hedging; Volatility smiles; Stochastic volatility

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## 1. Introduction

Since Black and Scholes (1973) published their seminal article on option pricing, there has been much theoretical and empirical work on option pricing. Numerous empirical studies have found that the Black–Scholes model (henceforth BS) results in systematic biases across moneyness and maturity. It is well known that after the October 1987 crash, the implied volatility computed from options on the stock index in the US market inferred from BS appears to be different across exercise prices. This is the so-called "volatility smiles". Of course, given BS assumptions, all option prices on the same underlying security with the same expiration date but with different exercise prices should have the same implied volatility. However, the "volatility smiles" pattern suggests that BS tends to misprice deep in-the-money and deep out-of-the-money options.[1]

There have been various attempts to deal with this apparent failure of BS. One important direction along which the BS formula can be modified is to generalize the geometric Brownian motion that is used as a model for the dynamics of log stock prices. For example, Hull and White (1987), Johnson and Shanno (1987), Scott (1987), Wiggins (1987), Melino and Turnbull (1990, 1995), Stein and Stein (1991) and Heston (1993) suggest a continuous-time stochastic volatility model. Merton (1976), Bates (1991) and Naik and Lee (1990) propose a jump-diffusion model. Duan (1995) and Heston and Nandi (2000) develop an option pricing model based on the GARCH process. Recently, Madan et al. (1998) use a three-parameter stochastic process, termed the variance gamma process, as an alternative model for the dynamics of log stock prices.

This wide range of stochastic volatility models that account for non-constant volatility requires comparison. Previously, Bakshi et al. (1997) evaluated the performance of alternative models for the S&P 500 index option contracts. They examined how much each additional feature improves the pricing and hedging performance. They showed that the stochastic volatility term provides a first-order improvement over BS. In addition, other factors such as the stochastic interest rate or the jump diffusion have a marginal effect.[2] To this end, we have a horse race competition among alternative stochastic volatility models to gauge pricing and hedging performance.

Thus we are comparing the relative empirical performance of four classes of stochastic volatility option pricing models. The first class of stochastic volatility models is the ad hoc Black and Scholes procedure (henceforth AHBS) dealt by Dumas et al. (1998). Assuming that option prices are given for all strikes and for all maturities, AHBS fits a volatility function for the underlying asset price process to the prices of option contracts. Once the volatility function is determined, it can be used to price and hedge other derivative assets.

The second class of stochastic volatility models is the GARCH type option pricing model (henceforth GARCH) of Heston and Nandi (2000). The autoregressive structure of the GARCH process captures empirical appearances like volatility clustering, leptokurtic return distributions and leverage effects. We choose this model because it yields the closed form solution.

[1] In-the-money, at-the-money and out-of-the-money is henceforth ITM, ATM and OTM, respectively.

[2] For the KOPSI 200 index option, Jung (2001) showed the same result as Bakshi et al. (1997).

The third class of stochastic volatility models is the variance gamma option pricing model (henceforth VG). The variance gamma process derived by Madan and Milne (1991) is aimed at providing a model for a log-return distribution that offers physical interpretation and incorporates both long-tailness and skewness characteristics in a log-return distribution. Using this process, Madan et al. (1998) derived the closed-form solution of the European call option.

The fourth class of stochastic volatility models is the continuous-time stochastic volatility model (henceforth SV) of Heston (1993) which models the square of the volatility process with mean-reverting dynamics, allowing for changes in the underlying asset price to be contemporaneously correlated with changes in the volatility process. We choose this model among other continuous-time stochastic models because of the allowance of the correlation between asset returns and volatility, and it yields the closed form solution.

Moreover, we compare alternative stochastic volatility option pricing models with the simplest but still valuable option pricing model, BS.

This study fills two gaps. First, this study considered improvements over BS by allowing stochastic volatility terms in pricing the KOSPI 200 index options. Although there are several studies that have examined the performance of the stochastic volatility option pricing models in major markets, such as S&P 500 and FTSE 100, no study has investigated their performance in emerging markets like the Korean options market. We doubt whether the stochastic volatility model exhibits an effective value in emerging markets. Most market practitioners in emerging markets still use BS, and markets reflecting this viewpoint can show that BS does not render such bad results in either pricing or hedging performances. Moreover, an important point mentioned by Bakshi et al. (1997) is "The volatility smiles are the strongest for short-term options (both calls and puts), indicating that short-term options are the most severely mispriced by BS and present perhaps the greatest challenge to any alternative option pricing model". Thus the KOSPI 200 index options market with liquidity, which is concentrated in the nearest expiration contract, will be an excellent sample market to investigate mispricing of short-term options. Second, while there are several papers that compare the incremental contribution of the stochastic volatility or the jump diffusion in explaining option pricing biases, there is a paucity of studies that compare alternative stochastic volatility option pricing models.

It has been found that SV outperforms other models in the in-sample, out-of-sample and hedging. Looking at the valuation errors by moneyness, the pricing and hedging errors are highest for OTM options and decrease as we move to ITM options in all models. The stochastic volatility models cannot mitigate the "volatility smiles" effects found in cross-sectional options data, but they can reduce the effects better than BS. GARCH shows the worst performance, but the performance of BS is not too far behind the stochastic volatility option pricing models.

The outline of this paper is as follows. Alternative stochastic volatility option pricing models are reviewed in Section 2. The data used for analysis are described in Section 3. Section 4 describes estimation methods. Section 5 describes parameter estimates of each model and evaluates pricing and hedging performances of alternative models. Section 6 concludes our study by summarizing the results.

## 2. Model

According to the option pricing theory, European options are priced by evaluating the expectation of the discounted terminal payoff of the option at maturity under an equivalent risk neutral measure $Q$. Hence the price of a European call with a strike price of $K$ and maturity $\tau$ is given by

$$C(t,\tau;K) = \mathrm{e}^{-r\tau}E_t^Q[\max(S_{t+\tau}-K,0)] \tag{1}$$

where $E_t^Q$ [·] represents the conditional expectation under the risk-neutral density.

Bakshi and Madan (2000) show that Eq. (1) can be decomposed into two components as

$$C = SP_1 - K\mathrm{e}^{-r\tau}P_2 \tag{2}$$

where

$$P_1 = E_t^Q\left[\frac{S_{t+\tau}}{S_t}1_{[S_{t+\tau}>K]}\right],$$

$$P_2 = E_t^Q\left[1_{[S_{t+\tau}>K]}\right]$$

and the indicator function $1_{[S_{t+\tau}}>K]$ is a unity when $S_{t+\tau}>K$. The price of a European put can be determined from the put–call parity.

In the rest of this section, we display only the probability $P_1$ and $P_2$ of each model.

### 2.1. AHBS

Since GARCH, VG and SV have more parameters than BS, they may have an unfair advantage over BS. Therefore, we follow Dumas et al. (1998) and construct AHBS in which each option has its own implied volatility depending on a strike price and time to maturity. Specifically, the spot volatility of the asset that enters BS is a function of the strike price and the time to maturity or a combination of both. However, we consider only the function of the strike price because the liquidity of the KOPSI 200 index options market is concentrated in the nearest expiration contract. Even if there are options with multiple maturities in a specific day, only the function of the strike price is applied. This is because parameters of that day must be plugged into the following day's data, and that data may have options with a single maturity in out-of-sample pricing and hedging.

Specifically we adopt the following specification for the BS implied volatilities:

$$\sigma_n = \beta_1 + \beta_2(S/K_n) + \beta_3(S/K_n)^2 \tag{3}$$

where $\sigma_n$ is the implied volatility for an $n$th option of strike $K_n$ and spot price $S$.

We follow a four-step procedure. First, we abstract the BS implied volatility from each option. Second, we estimate the $\beta_i\,(i=1,\ 2,\ 3)$ by ordinary least squares. Third, using estimated parameters from the second step, we plug in each option's moneyness into the equation, and obtain the model-implied volatility for each option. Finally, we use volatility estimates computed in the third step to price options with the BS formula.

AHBS, although theoretically inconsistent, can be a more challenging benchmark than the simple BS for any competing option valuation model.

### *2.2. GARCH*

The importance of the GARCH option pricing model has recently expanded due to its linkage with continuous-time stochastic models that are difficult to implement. The volatility of continuous-time stochastic models is not readily identifiable with discrete observations on the underlying asset price process. In contrast, GARCH models have the advantage that volatility is observable from the history of underlying asset prices.

Other existing GARCH models do not have closed-form solutions for option values. These models are typically solved by simulation (Engle and Mustafa, 1992; Amin and Ng, 1993; Duan, 1995) that require slow and computationally intensive empirical work. In contrast, Heston and Nandi (2000) develop a closed form solution for European option values and hedge ratios.

Under risk-neutral dynamics, the single lag version of their model takes the following form:

$$\ln\left[\frac{S_{t+1}}{S_t}\right] = r - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}Z_{t+1}, \tag{4}$$

$$h_{t+1} = \omega + \beta h_t + \alpha(Z_t - \gamma\sqrt{h_t})^2. \tag{5}$$

They derive risk neutral probabilities of European call option prices in a closed form, assuming that the value of a call option with one period to expiration obeys the BS formula as follows:

$$P_1 = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} Re\left[\frac{e^{-i\phi\ln[K]}f(\phi - i)}{i\phi}\right]d\phi, \tag{6}$$

$$P_2 = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} Re\left[\frac{e^{-i\phi\ln[K]}f(\phi)}{i\phi}\right]d\phi, \tag{7}$$

where $Re[\cdot]$ denotes the real part of complex variables, $i$ is the imaginary number, $\sqrt{-1}$, $f(\phi) = \exp(A(t;T,\phi) + B(t;T,\phi)h_{t+1} + i\phi\ln[S_t])$, $A(t;\tau,\phi)$ and $B(t;\tau,\phi)$ are functions of $\alpha$, $\beta$, $\gamma$ and $\omega$.

### *2.3. VG*

The variance gamma approach proposed by Madan and Seneta (1990), Madan and Milne (1991) has the advantage that additional parameters in the variance gamma process provide control over the skewness and kurtosis of the return distribution.

The variance gamma process is obtained by evaluating Brownian motion with drift at a random time given by a gamma process. Let

$$b(t;\theta,\sigma) = \theta t + \sigma W(t) \tag{8}$$

where $W(t)$ is a standard Brownian motion. The process $b(t;\ \theta,\ \sigma)$ is a Brownian motion with drift $\theta$ and volatility $\sigma$. The gamma process $\gamma(t;\ \mu,\ v)$ with mean rate $\mu$ and variance rate $v$ is the process of independent gamma increments over non-overlapping intervals. VG process, $X(t;\ \sigma,\ v,\ \theta)$, is defined in terms of the Brownian motion with drift $b(t;\ \theta,\ \sigma)$ and the gamma process with unit mean rate, $\gamma(t;\ 1,\ v)$ as $X(t;\ \sigma,\ v,\ \theta) = b(\gamma(t;\ 1,\ v),\ \theta,\ \sigma)$.

Thus, the assumed process of the underlying asset, $S_t$, is given by replacing the role of Brownian motion in the original Black–Scholes geometric Brownian motion model by the variance gamma process as follows:

$$S_t = S_0\ \exp[mt + X(t;\sigma,v,\theta) + wt] \tag{9}$$

where $S_0$ is the initial stock price, $m$ is the mean rate of stock return, and $w = (1/v)\ln(1 - \theta v - \sigma^2 v/2)$.

Based on the above process, Madan et al. (1998) derive risk neutral probabilities for the price of a European option as follows:

$$P_1 = \varphi\left[\mathrm{d}\sqrt{\frac{1-c_1}{v}}, (\alpha+\sigma)\sqrt{\frac{v}{1-c_1}}, \frac{\tau}{v}\right], \tag{10}$$

$$P_2 = \varphi\left[\mathrm{d}\sqrt{\frac{1-c_2}{v}}, \alpha\sqrt{\frac{v}{1-c_2}}, \frac{\tau}{v}\right], \tag{11}$$

where $\varphi(a,b,\gamma) = \int_0^\infty \Phi\left(\frac{a}{\sqrt{g}} + b\sqrt{g}\right)\frac{g^{\gamma-1}\mathrm{e}^{-g}}{\Gamma(\gamma)}\mathrm{d}g$, $\Phi(\cdot)$ is the cumulative distribution function of a standard normal distribution, and $\Gamma(\cdot)$ is the gamma function.

### 2.4. SV

Heston (1993) provided a closed-form solution for pricing a European style option when volatility follows a mean-reverting square-root process. The actual diffusion processes for the underlying asset and its volatility are specified as

$$\mathrm{d}S = \mu S\mathrm{d}t + \sqrt{v_t}S\mathrm{d}W_S, \tag{12}$$

$$\mathrm{d}v_t = \kappa(\theta - v_t)\mathrm{d}t + \sigma\sqrt{v_t}\mathrm{d}W_v, \tag{13}$$

where $\mathrm{d}W_S$ and $\mathrm{d}W_v$ have an arbitrary correlation $\rho$, $v_t$ is the instantaneous variance. $\kappa$ is the speed of adjustment to the long-run mean $\theta$, and $\sigma$ is the variation coefficient of variance.

Given the dynamics in Eqs. (12) and (13), Heston (1993) shows that risk neutral probabilities of a European call option with $\tau$ periods to maturity is given by

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} Re\left[\frac{\mathrm{e}^{-i\phi \ln[K]} f_j(x, v_t, \tau; \phi)}{i\phi}\right]\mathrm{d}\phi \quad (j = 1, 2) \tag{14}$$

where $Re[\cdot]$ denotes the real part of complex variables, $i$ is the imaginary number, $\sqrt{-1}$, $f_j(x,v_t,\tau;\phi) = \exp[C(\tau;\phi) + D(\tau;\phi)v_t + i\phi x]$ and $C(\tau;\phi)$ and $D(\tau;\phi)$ are functions of $\theta$, $\kappa$, $\rho$, $\sigma$ and $v_t$.

Because BS is already generally known, we do not display it separately.

## 3. Data

In July 7 1997, the Korean exchange for options introduced the KOSPI 200 index options. The KOSPI 200 options market has become one of the fastest growing markets in the world, despite its short history. Its daily trading volume reached 1.2 million contracts in November 2000, marking the most active index options product internationally.

Three consecutive near-term delivery months and one additional month from the quarterly cycle (March, June, September and December) make up four contract months. The expiration day is the second Thursday of each contract month. Each options contract month has at least five strike prices. The number of strike prices may, however, increase according to the price movement. Trading in the KOSPI 200 index options is fully automated. The exercise style of the KOSPI 200 options is European and thus contracts can be exercised only on the expiration dates. Hence our test results are not affected by the complication that arises due to the early exercise feature of American options. Moreover, it is important to note that liquidity is concentrated in the nearest expiration contract.

The sample period extends from January 3, 1999 through December 26, 2000. The minute-by-minute transaction prices for the KOSPI 200 options are obtained from the Korea Stock Exchange. The 3-month treasury yields were used as risk-free interest rates.[3] Because KOSPI 200 contracts are European-style, index levels were adjusted for dividend payments before each option's expiration date. The KOSPI 200 index pays dividends only at the end of March, June, September and December, which are used for adjustment dates.[4]

The following rules are applied to filter data needed for the empirical test.

1. For each day in the sample, only the last reported transaction price, which has to occur between 2:30 and 3:00 p.m., of each option contract is employed in the empirical test.

[3] Korea does not have a liquid Treasury bill market, so the 3-month Treasury yield is used in spite of the mismatch of maturity of options and interest rates.

[4] We assume that traders have perfect knowledge about future dividend payments because options in this study have short time-to-maturities.

Table 1
KOSPI 200 options data

| $S/K$ | 1999 | | 2000 | | All | |
|---|---|---|---|---|---|---|
| | Call | Put | Call | Put | Call | Put |
| < 0.94 | 1.80 (584) | 11.45 (268) | 1.41 (982) | 12.75 (600) | 1.55 (1566) | 12.35 (868) |
| 0.94–0.97 | 3.07 (309) | 6.48 (253) | 2.57 (333) | 6.33 (292) | 2.81 (642) | 6.40 (545) |
| 0.97–1.00 | 4.19 (308) | 4.80 (295) | 3.41 (318) | 4.53 (309) | 3.79 (626) | 4.66 (604) |
| 1.00–1.03 | 5.46 (273) | 3.38 (282) | 4.72 (255) | 3.23 (289) | 5.10 (528) | 3.30 (571) |
| 1.03–1.06 | 7.13 (204) | 2.27 (266) | 6.17 (210) | 2.30 (284) | 6.64 (414) | 2.28 (550) |
| ≥ 1.06 | 12.06 (382) | 1.36 (789) | 9.36 (258) | 1.39 (619) | 10.97 (640) | 1.37 (1408) |
| All | 5.26 (2060) | 4.06 (2153) | 3.49 (2356) | 5.58 (2393) | 4.32 (4416) | 4.86 (4546) |

This table reports average option price, and the number of options, which are shown in parentheses, for each moneyness, type (call or put) category. The sample period is from January 3, 1999 to December 26, 2000. Daily information from the last transaction prices (prior to 3:00 p.m.) of each option contract is used to obtain the summary statistics. Moneyness is defined as $S/K$, where $S$ denotes the spot price and $K$ denotes the strike price.

The tight time window is chosen to minimize problems stemming from intra-day variation in volatility.[5]

2. An option of a particular moneyness and maturity is represented only once in the sample. In other words, although the same option may be quoted again during the time window, only the last record of that option is included in our sample.
3. As options with less than 6 days and more than 90 days, to expiration may induce liquidity-related biases, they are excluded from the sample.
4. To mitigate the impact of price discreteness on option valuation, prices lower than 0.5 are not included.
5. Prices not satisfying the arbitrage restriction are excluded:

$$C_{t,\tau} \geq S_t - \sum_{s=1}^{\tau} e^{-r_{t,s}s} D_{t+S} - KB_{t,\tau},$$

$$P_{t,\tau} \geq KB_{t,\tau} - S_t + \sum_{s=1} e^{-r_{t,s}s} D_{t+s},$$

where $B_{t,\tau}$ is a zero-coupon bond that pays 1 in $\tau$ periods from time $t$ and $D_t$ is daily dividends in time $t$, $r_{t,s}$ the risk-free interest rate with maturity $s$ at date $t$.

We divide the option data into several categories according to the moneyness, $S/K$. Table 1 describes certain sample properties of the KOSPI 200 option prices used in the study. Summary statistics are reported for the option price and the total number of observations, according to each moneyness-option type category. Because the liquidity of the KOSPI 200 option contracts is concentrated in the nearest expiration contract, we do not observe the maturity category separately. Note that there are 4416 call and 4546 put option observations, with deep OTM options respectively taking up 35% for call and 31% for put.

[5] Because the recorded KOPSI 200 index values are not the daily closing index levels, there is no non-synchronous price issue here, except that the KOSPI 200 index level itself may contain stale component stock prices at each point in time.

Table 2
Implied volatility

| | $S/K$ | $<0.94$ | 0.94–0.97 | 0.97–1.00 | 1.00–1.03 | 1.03–1.06 | $\geq 1.06$ |
|---|---|---|---|---|---|---|---|
| Jan. 1999–June 1999 | Call | 0.5494 | 0.5370 | 0.5487 | 0.5476 | 0.5824 | 0.7517 |
| | Put | 0.4814 | 0.4640 | 0.4740 | 0.4868 | 0.4940 | 0.5284 |
| July 1999–Dec. 1999 | Call | 0.5019 | 0.4806 | 0.4750 | 0.4757 | 0.5022 | 0.6245 |
| | Put | 0.5311 | 0.4251 | 0.4387 | 0.4491 | 0.4393 | 0.5089 |
| Jan. 2000–June 2000 | Call | 0.4230 | 0.4207 | 0.4106 | 0.4136 | 0.4137 | 0.4423 |
| | Put | 0.5240 | 0.4258 | 0.4161 | 0.4208 | 0.4250 | 0.4657 |
| July 2000–Dec. 2000 | Call | 0.5080 | 0.4903 | 0.4806 | 0.4813 | 0.4879 | 0.5509 |
| | Put | 0.6440 | 0.4944 | 0.5033 | 0.4968 | 0.5154 | 0.5302 |

This table reports the implied volatilities calculated by inverting the Black–Scholes model separately for each moneyness category. The implied volatilities of individual options are then averaged within each moneyness category and across the days in the sample. Moneyness is defined as $S/K$, where $S$ denotes the spot price and $K$ denotes the strike price.

Table 2 presents the "volatility smiles" effects for four consecutive subperiods. We employ six fixed intervals for the degree of moneyness, and compute the mean over alternative subperiods of the implied volatility. It is interesting to note that the Korean options market seems to be "smiling" independent of the subperiods employed in the estimation. So, we recognize the need of the stochastic volatility option pricing model to mitigate this effect.

## 4. Estimation procedure

In applying option pricing models, one always encounters the difficulty that spot volatility and structural parameters are unobservable. We follow the estimation method similar to standard practices (e.g. Bakshi et al., 1997, 2000; Bates, 1991, 2000; Kirgiz, 2001; Lin et al., 2001; Nandi, 1998), and estimate parameters of each model every sample day.

Since closed-from solutions are available for an option price, a natural candidate for the estimation of parameters which enter the pricing and hedging formula is a non-linear least squares procedure involving minimization of the sum of squared errors between the model and market prices.

Estimating parameters from the physical asset returns can be an alternative, but historical data reflect only what happened in the past. Further, the procedure using historical data is not able to identify volatility risk premiums that have to be estimated from the options data conditional on the estimates of other parameters. The important advantage of using option prices to estimate parameters is to allow one to use the forward-looking information contained in the option prices.

Let $O_i(t, \tau; K)$ denote the market price of option $i$ on day $t$, and let $O_i^*(t, \tau; K)$ denote the model price of the option $i$ on day $t$. To estimate parameters of each model, we minimize the sum of squared percentage errors between model and market prices:

$$\min_{\phi_t} \sum_{i=1}^{N} \left[ \frac{O_i(t, \tau; K) - O_i^*(t, \tau; K)}{O_i(t, \tau; K)} \right]^2 \quad (t = 1, \ldots, T) \tag{15}$$

where $N$ denotes the number of options on day $t$, and $T$ denotes the number of days in the sample. Conventionally, the objective function to minimize the sum of squared errors is used. But we adopt the above function because the fit for OTM options under the conventional method that gives more weight to relatively expensive ITM options is the worst.[6]

For BS, the volatility parameter, $\sigma$, is estimated. For GARCH, we estimate the structural parameters, $\{\alpha, \beta, \gamma, \omega\}$ and conditional variance, $h_{t+1}$, is not estimated as an additional parameter but determined from the daily history of index returns and the structural parameters on day $t$. The starting variance, $h_0$, is the estimate of the variance for the past 1 year, computed from daily logarithmic returns. For VG, the unobservable volatility parameter $v$ with structural parameters $\{\alpha, \sigma\}$ is estimated. For SV, we estimate the unobservable volatility parameter $v_t$ with structural parameters $\{\theta, \kappa, \rho, \sigma\}$. As mentioned before, the coefficients for AHBS are estimated via ordinary least squares.

## 5. Empirical findings

In this section, we compare empirical performances of alternative stochastic volatility models with respect to three metrics: (1) in-sample performance, (2) out-of-sample performance, and (3) hedging performance.

The analysis is based on four measures: mean absolute errors (henceforth MAEs), mean percentage errors (henceforth MPEs), mean absolute percentage errors (henceforth MAPEs), and mean squared errors (henceforth MSEs). MAEs and MAPEs measure the magnitude of pricing errors, while MPEs indicate the direction of the pricing errors. MSEs measure the volatility of errors. In the remaining sections, we mainly deal with MAPEs, because the relative comparison considering each option price is important above all else.

### *5.1. In-sample pricing performance*

For each model, Table 3 reports average and standard deviations (in parentheses) of parameters, which are estimated daily. The implicit parameters are not constrained to be constant over time. While re-estimating the parameters daily is admittedly potentially inconsistent with the assumption of constant or slow-changing parameters used in deriving the option pricing model, such estimation is useful for indicating market sentiment on a daily basis.

Parameters of other stochastic volatility models except GARCH have large standard deviations. This shows that the stability of parameters is not supported for each model. However, as stated thereafter, the pricing performance of the model with parameters having large standard deviations is better than that of the model with parameters having

[6] There was no large difference between results using the sum of squared errors and those using the sum of squared percentage errors in our sample.

Table 3
Parameter estimates

| BS | | AHBS | | VG | | GARCH | | SV | |
|---|---|---|---|---|---|---|---|---|---|
| $\sigma$ | 0.4712 | $\beta_1$ | 2.3432 | $\alpha$ | 2.2670 | $\alpha$ | 2.98e−6 | $\theta$ | 2.8375 |
| | (0.0880) | | (5.4914) | | (6.7975) | | (1.81e − 7) | | (9.8256) |
| | | $\beta_2$ | − 3.8371 | $\sigma$ | 0.4310 | $\beta$ | 0.6420 | $\kappa$ | 9.5723 |
| | | | (10.943) | | (0.1391) | | (0.0595) | | (29.9060) |
| | | $\beta_3$ | 1.9693 | $\nu$ | 0.0311 | $\gamma$ | 316.2083 | $\rho$ | − 0.0348 |
| | | | (5.4493) | | (0.0836) | | (23.6614) | | (0.7867) |
| | | | | | | $\omega$ | 7.00e − 6 | $\sigma$ | 0.5674 |
| | | | | | | | (4.57e − 6) | | (0.3040) |
| | | | | | | | | $\nu$ | 0.2856 |
| | | | | | | | | | (0.3456) |

The table reports the mean and standard deviation (in parentheses) of the parameter estimates for each model. BS is the Black–Scholes model in which a single implied volatility is estimated across all strikes and maturities on a given day. AHBS is the ad hoc BS procedure in which regression specification is estimated as follows:

$$\sigma_n = \beta_1 + \beta_2 S/K_n + \beta_3 (S/K_n)^2.$$

VG, GARCH and SV are Madan et al.'s (1998) variance gamma model, Heston and Nandi's (2000) GARCH type discrete model and Heston's (1993) continuous-time stochastic volatility model in which each parameter is estimated by minimizing the sum of percentage squared errors between model and market option prices every day.

small ones, i.e. it is found that the stability of interdependence among parameters is more important than that of parameters in option pricing and hedging.

The implied correlation coefficient is negative as we expected.[7] This is consistent with the leverage effect documented by Black (1976) and Christie (1982), whereby lower overall firm values increase the volatility of equity returns, and the volatility feedback effects of Poterba and Summers (1986) whereby higher volatility assessments lead to heavier discounting of future expected dividends and thereby lower equity price.

We evaluate the in-sample performance of each model by comparing market prices with model prices computed by using the parameter estimates from the current day. Table 4 reports the in-sample valuation errors for alternative models computed over the whole sample of options as well as across six moneyness and two option type categories. Results from the analysis are as follows.

First, with respect to MAPEs and MAEs, SV shows the best performance followed by VG. However, according to MSEs, the order has been changed. For call options, AHBS has the fewest errors followed by SV. For put options, BS has the best performance followed by SV. On the whole, SV is the best for the in-sample pricing.

Unexpectedly, AHBS is not better than BS although AHBS has more parameters than BS does. This result can be explained by the lower $R^2$ compared to advanced markets. In our study, the $R^2$ of AHBS is 22% on average, which is quite low. In the study of Kirgiz

[7] The positive $\alpha$ and $\gamma$ of VG and GARCH indicate a negative correlation, respectively.

(2001) on the S&P 500, the $R^2$ was 93%. Because of the low $R^2$, AHBS seems to lead to a relatively large in-sample errors.

In the moneyness-based error, for call options, SV has the fewest errors except deep ITM options where AHBS does. For put options, AHBS in deep ITM, GARCH in ITM

Table 4
In-sample pricing errors

| | S/K | < 0.94 | 0.94–0.97 | 0.97–1.00 | 1.00–1.03 | 1.03–1.06 | ≥ 1.06 | All |
|---|---|---|---|---|---|---|---|---|
| *Panel A: Calls* | | | | | | | | |
| MPE | BS | 0.0305 | 0.0441 | 0.0159 | 0.0066 | 0.0150 | 0.0258 | 0.0254 |
| | AHBS | − 0.1176 | 0.0283 | 0.0099 | 0.0000 | 0.0018 | 0.0063 | − 0.0351 |
| | GARCH | 0.2000 | 0.0864 | 0.0253 | − 0.0031 | − 0.0024 | 0.0141 | 0.0885 |
| | VG | − 0.0218 | 0.0386 | 0.0364 | 0.0280 | 0.0275 | 0.0275 | 0.0130 |
| | SV | 0.0280 | 0.0046 | 0.0027 | 0.0104 | 0.0207 | 0.0280 | 0.0188 |
| MAPE | BS | 0.1635 | 0.1538 | 0.1269 | 0.1049 | 0.0843 | 0.0640 | 0.1280 |
| | AHBS | 0.2687 | 0.1640 | 0.1230 | 0.0934 | 0.0700 | 0.0457 | 0.1609 |
| | GARCH | 0.2497 | 0.1920 | 0.1508 | 0.1215 | 0.0947 | 0.0672 | 0.1710 |
| | VG | 0.1112 | 0.1206 | 0.1201 | 0.1152 | 0.0944 | 0.0667 | 0.1063 |
| | SV | 0.0808 | 0.1126 | 0.1086 | 0.0989 | 0.0856 | 0.0666 | 0.0899 |
| MAE | BS | 0.2572 | 0.4344 | 0.4659 | 0.5415 | 0.5780 | 0.7230 | 0.4442 |
| | AHBS | 0.3552 | 0.4269 | 0.4384 | 0.4704 | 0.4734 | 0.4904 | 0.4219 |
| | GARCH | 0.3651 | 0.5471 | 0.5582 | 0.6213 | 0.6423 | 0.7439 | 0.5305 |
| | VG | 0.1810 | 0.3426 | 0.4422 | 0.5921 | 0.6445 | 0.7447 | 0.4158 |
| | SV | 0.1380 | 0.3227 | 0.4006 | 0.5038 | 0.5771 | 0.7433 | 0.3752 |
| MSE | BS | 0.1806 | 0.4924 | 0.4207 | 0.5974 | 0.7072 | 1.5614 | 0.5593 |
| | AHBS | 0.3632 | 0.4035 | 0.3602 | 0.4517 | 0.4953 | 0.7772 | 0.4516 |
| | GARCH | 0.3038 | 0.7132 | 0.5664 | 0.7362 | 0.8379 | 1.5856 | 0.6883 |
| | VG | 0.1107 | 0.3469 | 0.3601 | 0.6929 | 0.8401 | 1.5922 | 0.5331 |
| | SV | 0.0734 | 0.3234 | 0.3203 | 0.4883 | 0.6611 | 1.5957 | 0.4716 |
| *Panel B: Puts* | | | | | | | | |
| MPE | BS | 0.0169 | − 0.0277 | − 0.0345 | − 0.0347 | − 0.0245 | 0.0692 | 0.0094 |
| | AHBS | 0.0051 | − 0.0312 | − 0.0402 | − 0.0507 | − 0.0839 | − 0.1538 | − 0.0722 |
| | GARCH | 0.0271 | − 0.0099 | − 0.0246 | − 0.0517 | − 0.0894 | − 0.0856 | − 0.0431 |
| | VG | 0.0128 | − 0.0307 | − 0.0189 | 0.0103 | 0.0381 | 0.0701 | 0.0239 |
| | SV | 0.0132 | − 0.0371 | − 0.0478 | − 0.0438 | − 0.0280 | 0.0343 | − 0.0066 |
| MAPE | BS | 0.0528 | 0.0858 | 0.1030 | 0.1235 | 0.1355 | 0.1651 | 0.1171 |
| | AHBS | 0.0396 | 0.0796 | 0.1029 | 0.1375 | 0.1856 | 0.2745 | 0.1555 |
| | GARCH | 0.0523 | 0.0791 | 0.0990 | 0.1327 | 0.1692 | 0.1855 | 0.1272 |
| | VG | 0.0581 | 0.0964 | 0.1008 | 0.0963 | 0.0941 | 0.1290 | 0.0995 |
| | SV | 0.0569 | 0.0911 | 0.1031 | 0.1096 | 0.0985 | 0.0851 | 0.0876 |
| MAE | BS | 0.6407 | 0.5286 | 0.4494 | 0.3726 | 0.2771 | 0.2159 | 0.3926 |
| | AHBS | 0.4561 | 0.4917 | 0.4490 | 0.4055 | 0.3438 | 0.3225 | 0.3981 |
| | GARCH | 0.6556 | 0.4985 | 0.4356 | 0.3904 | 0.3233 | 0.2350 | 0.4039 |
| | VG | 0.6901 | 0.5991 | 0.4530 | 0.3220 | 0.2192 | 0.1792 | 0.3863 |
| | SV | 0.6875 | 0.5624 | 0.4490 | 0.3318 | 0.2092 | 0.1230 | 0.3626 |
| MSE | BS | 1.0362 | 0.5103 | 0.3567 | 0.2447 | 0.1401 | 0.1251 | 0.3929 |
| | AHBS | 0.5089 | 0.4872 | 0.3571 | 0.2925 | 0.2239 | 0.6908 | 0.4808 |
| | GARCH | 1.1775 | 0.4643 | 0.3316 | 0.2693 | 0.1758 | 0.1487 | 0.4260 |
| | VG | 1.1371 | 0.6799 | 0.3774 | 0.2269 | 0.1160 | 0.1157 | 0.4272 |
| | SV | 1.1688 | 0.5667 | 0.3499 | 0.2009 | 0.0910 | 0.0787 | 0.3962 |

and ATM, VG in ATM and OTM and SV in deep OTM, show the best performance. But SV has the fewest errors on average, because the number of samples is concentrated in deep OTM. In the direction of pricing errors, there is no specific property except that BS undervalues call options for all moneyness.

Compared with the studies of advanced markets, BS shows good results. BS does not have the best performance but it makes quite a good fit considering that it uses only one parameter. The performance of GARCH is different than Heston and Nandi (2000), who find GARCH to perform better than AHBS. This different result can be explained by the difference of the data sets applied. They exclude options with deep ITM and deep OTM, but we include them because there is a large portion in the data set.

Second, all models show moneyness-based valuation errors, and exhibit the worst fit for the OTM options. The fit of the models, as measured by MAPEs, steadily decrease as we move from OTM to ITM options except that MAPEs of VG and SV increase to the second OTM and decrease after that. Although the objective function to minimize the percentage errors to mitigate moneyness-based errors is used, this bias continues.

To sum up, SV shows the best in-sample performance.

### 5.2. Out-of-sample pricing performance

The output of the in-sample performance can be biased due to the potential problem of over-fitting to the data. A good in-sample fit might be a consequence of having an increasingly larger number of structural parameters. To lower the impact of this connection on inferences, we turn to examining the model's out-of-sample cross-sectional pricing performance. In the out-of-sample pricing, the presence of more parameters may actually cause over-fitting, and have the model penalized if the extra parameters do not improve structural fitting. This analysis also has the purpose of assessing each model's parameter stability over time. To control for parameter stability over alternative time periods, we analyze out-of-sample valuation errors for the next day (week). We use the current day's estimated structural parameters to price options on the next day (week).

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Notes to Table 4:

This table reports in-sample pricing errors for the KOSPI 200 option with respect to moneyness. $S/K$ is defined as moneyness, where $S$ denotes the asset price and $K$ denotes the strike price. Each model is estimated every day during the sample period and in-sample pricing errors are computed using parameters estimated from the current day. Denoting $\varepsilon_n = O_n - O_n^*$, where $O_n$ is the market and $O_n^*$ is the model price, pricing performance is evaluated by: (1) mean percentage error (MPE), $(\sum_{n=1}^{N} \varepsilon_n / O_n)/N$, (2) mean absolute percentage error (MAPE), $(\sum_{n=1}^{N} |\varepsilon_n| / O_n)/N$, (3) mean absolute error (MAE), $(\sum_{n=1}^{N} |\varepsilon_n|)/N$, and (4) mean squared error (MSE), $(\sum_{n=1}^{N} (\varepsilon_n)^2)/N$, where $N$ is the total number of options in a particular moneyness category. BS denotes the Black and Scholes model, AHBS denotes the ad hoc Black and Scholes procedure that fits to the implied volatility surface, GARCH denotes Heston and Nandi's (2000) GARCH type discrete model, VG denotes Madan et al.'s (1998) variance gamma model, and SV denotes Heston's (1993) continuous-time stochastic volatility model.

For the other models except GARCH, the current day's estimated instantaneous volatility and structural parameters are used to price options for the next day (week). For GARCH, a conditional variance $h_{t+1}$ is computed by iterating the volatility

Table 5
One day ahead out-of-sample pricing errors

| | *S/K* | < 0.94 | 0.94–0.97 | 0.97–1.00 | 1.00–1.03 | 1.03–1.06 | ≥ 1.06 | All |
|---|---|---|---|---|---|---|---|---|
| *Panel A: Calls* | | | | | | | | |
| MPE | BS | 0.0281 | 0.0339 | 0.0115 | 0.0036 | 0.0133 | 0.0245 | 0.0217 |
| | AHBS | − 0.1042 | 0.0231 | 0.0073 | − 0.0040 | 0.0025 | 0.0074 | − 0.0318 |
| | GARCH | 0.2113 | 0.1253 | 0.0217 | − 0.0072 | − 0.0057 | 0.0352 | 0.0999 |
| | VG | − 0.0320 | 0.0302 | 0.0332 | 0.0281 | 0.0281 | 0.0267 | 0.0077 |
| | SV | 0.0270 | 0.0152 | 0.0014 | − 0.0061 | 0.0064 | 0.0190 | 0.0144 |
| MAPE | BS | 0.2205 | 0.1755 | 0.1384 | 0.1103 | 0.0875 | 0.0633 | 0.1540 |
| | AHBS | 0.3438 | 0.1766 | 0.1332 | 0.1031 | 0.0822 | 0.0584 | 0.1951 |
| | GARCH | 0.3018 | 0.2025 | 0.1612 | 0.1324 | 0.1050 | 0.0749 | 0.1958 |
| | VG | 0.1955 | 0.1549 | 0.1364 | 0.1189 | 0.0950 | 0.0656 | 0.1437 |
| | SV | 0.1923 | 0.1512 | 0.1259 | 0.1080 | 0.0882 | 0.0667 | 0.1383 |
| MAE | BS | 0.3219 | 0.4861 | 0.5088 | 0.5634 | 0.5954 | 0.7189 | 0.4842 |
| | AHBS | 0.4554 | 0.4816 | 0.4862 | 0.5204 | 0.5558 | 0.6642 | 0.5109 |
| | GARCH | 0.4668 | 0.5623 | 0.5689 | 0.7709 | 0.6944 | 0.7789 | 0.5981 |
| | VG | 0.2776 | 0.4189 | 0.5017 | 0.6101 | 0.6485 | 0.7356 | 0.4712 |
| | SV | 0.2788 | 0.4182 | 0.4660 | 0.5394 | 0.5933 | 0.7450 | 0.4554 |
| MSE | BS | 0.2400 | 0.5710 | 0.4872 | 0.6383 | 0.7186 | 1.5632 | 0.6070 |
| | AHBS | 0.7870 | 0.5864 | 0.4283 | 0.5270 | 0.6099 | 1.4003 | 0.7481 |
| | GARCH | 0.6547 | 0.7796 | 0.6823 | 0.8975 | 0.9625 | 2.0589 | 0.9382 |
| | VG | 0.1997 | 0.4549 | 0.5145 | 0.7521 | 0.8357 | 1.5791 | 0.6076 |
| | SV | 0.2376 | 0.5317 | 0.3991 | 0.5658 | 0.6765 | 1.6232 | 0.5868 |
| *Panel B: Puts* | | | | | | | | |
| MPE | BS | 0.0183 | − 0.0280 | − 0.0359 | − 0.0430 | − 0.0331 | 0.0554 | 0.0031 |
| | AHBS | 0.0090 | − 0.0315 | − 0.0397 | − 0.0600 | − 0.0850 | − 0.2404 | − 0.0995 |
| | GARCH | 0.0332 | − 0.0189 | − 0.0256 | − 0.0679 | − 0.0867 | − 0.1539 | − 0.0660 |
| | VG | 0.0132 | − 0.0306 | − 0.0214 | 0.0034 | 0.0309 | 0.0634 | 0.0196 |
| | SV | 0.0159 | − 0.0332 | − 0.0465 | − 0.0560 | − 0.0437 | 0.0206 | − 0.0133 |
| MAPE | BS | 0.0533 | 0.0903 | 0.1061 | 0.1364 | 0.1578 | 0.1982 | 0.1327 |
| | AHBS | 0.0512 | 0.0906 | 0.1096 | 0.1524 | 0.2010 | 0.4337 | 0.2129 |
| | GARCH | 0.0538 | 0.0972 | 0.1027 | 0.1467 | 0.1957 | 0.2846 | 0.1658 |
| | VG | 0.0586 | 0.1011 | 0.1059 | 0.1259 | 0.1427 | 0.1900 | 0.1291 |
| | SV | 0.0566 | 0.0910 | 0.1067 | 0.1334 | 0.1470 | 0.1802 | 0.1258 |
| MAE | BS | 0.6458 | 0.5567 | 0.4692 | 0.4156 | 0.3181 | 0.2513 | 0.4210 |
| | AHBS | 0.6208 | 0.5647 | 0.4871 | 0.4618 | 0.3823 | 0.4738 | 0.5020 |
| | GARCH | 0.6747 | 0.5956 | 0.4629 | 0.4923 | 0.3614 | 0.3814 | 0.4854 |
| | VG | 0.6938 | 0.6289 | 0.4777 | 0.4046 | 0.3007 | 0.2441 | 0.4351 |
| | SV | 0.6849 | 0.5636 | 0.4640 | 0.4044 | 0.2964 | 0.2263 | 0.4183 |
| MSE | BS | 1.0412 | 0.5674 | 0.3950 | 0.3003 | 0.1875 | 0.1526 | 0.4272 |
| | AHBS | 1.0149 | 0.6204 | 0.4374 | 0.3939 | 0.2752 | 1.3018 | 0.8121 |
| | GARCH | 1.2406 | 0.6118 | 0.3588 | 0.3809 | 0.2723 | 0.1799 | 0.4944 |
| | VG | 1.1300 | 0.7268 | 0.4215 | 0.3211 | 0.1785 | 0.1493 | 0.4688 |
| | SV | 1.1727 | 0.5712 | 0.3868 | 0.4287 | 0.2360 | 0.1924 | 0.4886 |

process up to the next day (week) using estimated structural parameters from the current day.

Tables 5 and 6 report 1 day and 1 week ahead out-of-sample valuation errors for alternative models computed over the whole sample of options, respectively. First, backed by each valuation measure, the relative ranking of the models gets changed. In 1 day ahead out-of-sample pricing, SV shows the best performance for all measures expect MSEs of put options, closely followed by BS and VG. In MSEs of put options, BS has the smallest errors followed by VG. In 1 week ahead out-of-sample pricing, for call options, SV and VG show better performance than other models with respect to MAPEs and MAEs. In MSEs, VG is the best followed by BS. For put options, SV shows the best performance closely followed by BS and VG. As a result, SV can be the best model in out-of-sample pricing, too.

Second, pricing errors deteriorate when shifting from the in-sample pricing to the out-of-sample pricing. The average of MAPEs of all models from a call (put) option is 13.12% (11.74%) for in-sample pricing, and grows to 16.54% (15.33%) for 1 day ahead out-of-sample pricing. There is not a striking contrast between the errors of in-sample pricing and 1 day ahead out-of-sample pricing. But, in 1 week ahead out-of-sample pricing errors, the errors grow to 21.40% (23.52%), which is double the in-sample pricing errors for call (put) options.

Third, the difference between BS and SV that show better performance rather than other models grows smaller in the out-of-sample pricing. The ratio of MAPEs from BS to SV is 1.424 (1.337) for in-sample errors of call (put) options. This ratio changes to 1.114 (1.050) and 1.021 (1.009) for 1 day and 1 week ahead out-of-sample errors, respectively. As the term of out-of-sample pricing gets longer, the difference between two models decreases. It is found that BS with a single parameter can have an advantage over other complicated models, especially in long-term forecasting.

Fourth, as in-sample pricing errors, out-of-sample pricing errors show moneyness-based biases. MAPEs decrease from OTM to ATM and then to ITM options for all models.

As was the case of in-sample pricing performances, BS exhibits a good fit for out-of-sample pricing contrary to performances in the advanced markets. This result shows BS is competent for out-of-sample pricing with the advantage of simplicity in the emerging

Notes to Table 5:

This table reports 1 day ahead out-of-sample pricing errors for the KOSPI 200 option with respect to moneyness. $S/K$ is defined as moneyness, where $S$ denotes the asset price and $K$ denotes the strike price. Each model is estimated every day during the sample period and 1 day ahead out-of-sample pricing errors are computed using parameters estimated the previous trading day. Denoting $\varepsilon_n = O_n - O_n^*$ where $O_n$ is the market and $O_n^*$ is the model price, pricing performance is evaluated by: (1) mean percentage error (MPE), $(\sum_{n=1}^{N} \varepsilon_n / O_n)/N$, (2) mean absolute percentage error (MAPE), $(\sum_{n=1}^{N} |\varepsilon_n| / O_n)/N$, (3) mean absolute error (MAE), $(\sum_{n=1}^{N} |\varepsilon_n|)/N$, and (4) mean squared error (MSE), $(\sum_{n=1}^{N} (\varepsilon_n)^2)/N$, where $N$ is the total number of options in a particular moneyness category. BS denotes the Black and Scholes model, AHBS denotes the ad hoc Black and Scholes procedure that fits to the implied volatility surface, GARCH denotes the Heston and Nandi's (2000) GARCH type discrete model, VG denotes the Madan et al.'s (1998) variance gamma model, and SV denotes Heston's (1993) continuous-time stochastic volatility model.

markets. Also, GARCH shows the worst performance irrespective of the rebalancing period except for put options in 1 day ahead pricing.

To further analyze the structure of out-of-sample pricing errors, we have a regression analysis that uses a combination of moneyness and interest rate as the

Table 6
One week ahead out-of-sample pricing errors

| | S/K | < 0.94 | 0.94 – 0.97 | 0.97 – 1.00 | 1.00 – 1.03 | 1.03 – 1.06 | ≥ 1.06 | All |
|---|---|---|---|---|---|---|---|---|
| *Panel A: Calls* | | | | | | | | |
| MPE | BS | 0.0337 | 0.0201 | − 0.0014 | − 0.0035 | 0.0114 | 0.0232 | 0.0187 |
| | AHBS | − 0.0745 | 0.0119 | − 0.0058 | − 0.0131 | − 0.0018 | 0.0108 | − 0.0258 |
| | GARCH | 0.1375 | 0.0409 | 0.0255 | − 0.0088 | − 0.0159 | 0.0276 | 0.0598 |
| | VG | − 0.0257 | 0.0128 | 0.0260 | 0.0248 | 0.0269 | 0.0264 | 0.0059 |
| | SV | 0.0523 | 0.0170 | − 0.0075 | − 0.0091 | 0.0086 | 0.0177 | 0.0217 |
| MAPE | BS | 0.2874 | 0.2042 | 0.1633 | 0.1225 | 0.0942 | 0.0641 | 0.1879 |
| | AHBS | 0.4767 | 0.2136 | 0.1671 | 0.1231 | 0.0973 | 0.0663 | 0.2579 |
| | GARCH | 0.4452 | 0.2536 | 0.2019 | 0.1563 | 0.1129 | 0.0716 | 0.2630 |
| | VG | 0.2583 | 0.1894 | 0.1645 | 0.1334 | 0.1017 | 0.0666 | 0.1773 |
| | SV | 0.2852 | 0.1961 | 0.1651 | 0.1233 | 0.0950 | 0.0644 | 0.1841 |
| MAE | BS | 0.4112 | 0.5409 | 0.5956 | 0.6263 | 0.6445 | 0.7258 | 0.5489 |
| | AHBS | 0.6360 | 0.5747 | 0.6134 | 0.6267 | 0.6633 | 0.7543 | 0.6423 |
| | GARCH | 0.5893 | 0.6296 | 0.6682 | 0.8136 | 0.7239 | 0.7639 | 0.6711 |
| | VG | 0.3623 | 0.4989 | 0.5974 | 0.6849 | 0.6996 | 0.7437 | 0.5418 |
| | SV | 0.3891 | 0.5126 | 0.5878 | 0.6258 | 0.6468 | 0.7216 | 0.5388 |
| MSE | BS | 0.3256 | 0.6029 | 0.6309 | 0.7222 | 0.7691 | 1.5765 | 0.6775 |
| | AHBS | 1.9257 | 0.7575 | 0.6947 | 0.7564 | 0.8071 | 1.7176 | 1.3067 |
| | GARCH | 0.8623 | 1.0696 | 1.0223 | 1.1346 | 0.9213 | 1.6833 | 1.0722 |
| | VG | 0.2649 | 0.5489 | 0.6245 | 0.8297 | 0.8721 | 1.6009 | 0.6764 |
| | SV | 0.3529 | 0.5740 | 0.6349 | 0.6989 | 0.7721 | 1.5691 | 0.6885 |
| *Panel B: Puts* | | | | | | | | |
| MPE | BS | 0.0211 | − 0.0272 | − 0.0444 | − 0.0539 | − 0.0473 | 0.0233 | − 0.0104 |
| | AHBS | 0.0111 | − 0.0313 | − 0.0531 | − 0.0760 | − 0.1036 | − 0.4151 | − 0.1593 |
| | GARCH | 0.0453 | − 0.0269 | − 0.0355 | − 0.0839 | − 0.1133 | − 0.2336 | − 0.0959 |
| | VG | 0.0156 | − 0.0304 | − 0.0243 | − 0.0003 | 0.0157 | 0.0378 | 0.0095 |
| | SV | 0.0233 | − 0.0265 | − 0.0454 | − 0.0594 | − 0.0484 | 0.0138 | − 0.0140 |
| MAPE | BS | 0.0593 | 0.1054 | 0.1347 | 0.1796 | 0.2181 | 0.2972 | 0.1828 |
| | AHBS | 0.0607 | 0.1124 | 0.1466 | 0.2042 | 0.2682 | 0.6806 | 0.3134 |
| | GARCH | 0.6108 | 0.1131 | 0.1255 | 0.1869 | 0.2529 | 0.3746 | 0.3170 |
| | VG | 0.0632 | 0.1134 | 0.1360 | 0.1743 | 0.2065 | 0.2969 | 0.1818 |
| | SV | 0.0612 | 0.1058 | 0.1350 | 0.1787 | 0.2131 | 0.2974 | 0.1812 |
| MAE | BS | 0.7140 | 0.6714 | 0.6117 | 0.5664 | 0.4651 | 0.3769 | 0.5423 |
| | AHBS | 0.7098 | 0.7163 | 0.6653 | 0.6372 | 0.5704 | 0.7394 | 0.6879 |
| | GARCH | 0.7229 | 0.6959 | 0.6553 | 0.6159 | 0.5569 | 0.5823 | 0.6336 |
| | VG | 0.7329 | 0.7203 | 0.6210 | 0.5608 | 0.4557 | 0.3750 | 0.5518 |
| | SV | 0.7296 | 0.6739 | 0.6047 | 0.5525 | 0.4469 | 0.3665 | 0.5397 |
| MSE | BS | 1.1710 | 0.8226 | 0.6529 | 0.5501 | 0.4082 | 0.3064 | 0.6226 |
| | AHBS | 1.1367 | 1.0308 | 0.8473 | 0.7136 | 0.6855 | 2.0204 | 1.2518 |
| | GARCH | 1.2339 | 0.7923 | 0.6422 | 0.6643 | 0.5834 | 0.5428 | 0.7380 |
| | VG | 1.0770 | 0.9735 | 0.6800 | 0.5513 | 0.3987 | 0.2982 | 0.6251 |
| | SV | 1.0738 | 0.8138 | 0.6298 | 0.5567 | 0.3679 | 0.2812 | 0.5922 |

explanatory variables. Among others, Madan et al. (1998) and Lam et al. (2002) have applied this regression for similar purposes. The mathematical expression of the regression model is

$$\varepsilon_{n.t} = \beta_0 + \beta_1(S_t/K_n) + \beta_2(S_t/K_n)^2 + \beta_3\tau_n + \beta_4 r_t + \eta_t \qquad (16)$$

where $\varepsilon_{n.t}$ denote the 1 day ahead absolute percentage pricing error on day $t$, $K_n$ is the strike price of the option, $\tau_n$ the time to maturity, and $r_t$ the risk-free interest rate at date $t$. The square of moneyness is employed to detect smile effects. A complete smile would result in a negative linear term and a positive quadratic term. The estimated value and $p$-values of each parameter are shown in Table 7.

In 1 day ahead out-of-sample pricing errors, moneyness variables are systematically related to MAPEs for all models. Coefficients of moneyness are significant both in linear and quadratic components showing a smile shape except that a linear component of BS and VG is not significant for put options. However, VG and SV show the best performance in a regression analysis for call and put option, respectively, because the adjusted $R^2$ coefficient and the $F$ statistics of VG and SV are the smallest. In 1 week ahead out-of-sample pricing errors, AHBS is the best. But the "volatility smiles" coefficients of AHBS are significant as well. All models considering the stochastic volatility show better performance rather than BS except VG for put options. This result indirectly shows that the addition of the stochastic volatility term does not settle the "volatility smiles" effects of BS but reduces the effects a little.

### 5.3. Hedging performance

Hedging performance is important to gauge the forecasting power of the volatility of underlying assets. We examine hedges in which only a single instrument (i.e. the underlying asset) can be employed. For the perfect replicating hedge in the context of a stochastic volatility model, one needs a position in (1) the underlying asset, (2) another option with the same maturity but different strike price, and (3) a riskless bond. However, in practice, option traders usually focus on the risk due to the

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Notes to Table 6:
This table reports 1 week ahead out-of-sample pricing errors for the KOSPI 200 option with respect to moneyness. $S/K$ is defined as moneyness, where $S$ denotes the asset price and $K$ denotes the strike price. Each model is estimated every day during the sample period and 1 week ahead out-of-sample pricing errors are computed using parameters estimated 1 week ago. Denoting $\varepsilon_n = O_n - O_n^*$, where $O_n$ is the market and $O_n^*$ is the model price, pricing performance is evaluated by: (1) mean percentage error (MPE), $(\sum_{n=1}^{N} \varepsilon_n/O_n)/N$, (2) mean absolute percentage error (MAPE), $(\sum_{n=1}^{N} |\varepsilon_n|/O_n)/N$, (3) mean absolute error (MAE), $(\sum_{n=1}^{N} |\varepsilon_n|)/N$, and (4) mean squared error (MSE), $(\sum_{n=1}^{N} (\varepsilon_n)^2)/N$, where $N$ is the total number of options in a particular moneyness category. BS denotes the Black and Scholes model, AHBS denotes the ad hoc Black and Scholes procedure that fits to the implied volatility surface, GARCH denotes Heston and Nandi's (2000) GARCH type discrete model, VG denotes Madan et al.'s (1998) variance gamma model, and SV denotes Heston's (1993) continuous-time stochastic volatility model.

Table 7
Regression coefficients of independent variables for pricing errors

| Coefficients | BS | AHBS | GARCH | VG | SV |
|---|---|---|---|---|---|
| Panel A: One day ahead out-of-sample pricing errors | | | | | |
| *Calls* | | | | | |
| $\beta_0$ | 1.7813 | 7.0386 | 2.0694 | 1.7595 | 1.6037 |
| | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.0000) |
| $\beta_1$ | − 3.3442 | − 13.110 | − 4.3369 | − 3.2812 | − 3.0952 |
| | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.0000) |
| $\beta_2$ | 1.3467 | 5.9329 | 1.8923 | 1.3768 | 1.2835 |
| | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.0000) |
| $\beta_3$ | 0.0727 | 0.5743 | 0.3496 | 0.1632 | − 0.0243 |
| | (0.3060) | (0.0054) | (0.0148) | (0.0518) | (0.7246) |
| $\beta_4$ | 5.0604 | 3.2280 | 7.3356 | 3.7985 | 4.8736 |
| | (0.0000) | (0.1168) | (0.0000) | (0.0000) | (0.0000) |
| Adjusted $R^2$ | 0.1399 | 0.0894 | 0.2669 | 0.0813 | 0.1158 |
| $F$ | 180.39 | 109.29 | 250.28 | 98.019 | 142.22 |
| *Puts* | | | | | |
| $\beta_0$ | − 0.1010 | 12.3519 | − 0.1623 | − 0.0762 | 0.4893 |
| | (0.4433) | (0.0000) | (0.0125) | (0.5614) | (0.0089) |
| $\beta_1$ | − 0.4655 | − 25.8267 | − 1.8396 | − 0.4555 | − 1.5802 |
| | (0.0596) | (0.0000) | (0.0000) | (0.0654) | (0.0000) |
| $\beta_2$ | 0.5387 | 13.5852 | 1.8463 | 0.5027 | 1.0442 |
| | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.0000) |
| $\beta_3$ | − 0.1186 | 1.0187 | 0.3462 | − 0.1060 | 0.1812 |
| | (0.0324) | (0.0013) | (0.0133) | (0.0564) | (0.0284) |
| $\beta_4$ | 2.2892 | − 1.7138 | 2.8843 | 2.2688 | 2.1419 |
| | (0.0000) | (0.5607) | (0.0000) | (0.0000) | (0.0033) |
| Adjusted $R^2$ | 0.1951 | 0.1562 | 0.2434 | 0.1691 | 0.1012 |
| $F$ | 276.12 | 211.16 | 321.29 | 230.45 | 126.522 |
| Panel B: 1 week ahead out-of-sample pricing errors | | | | | |
| *Calls* | | | | | |
| $\beta_0$ | 2.7392 | 11.3948 | 4.3589 | 1.9862 | 2.9133 |
| | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.0000) |
| $\beta_1$ | − 4.5999 | − 22.0588 | − 7.2239 | − 3.0818 | − 5.0266 |
| | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.0000) |
| $\beta_2$ | 1.8040 | 10.1403 | 2.6475 | 1.1216 | 2.0025 |
| | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.0000) |
| $\beta_3$ | − 0.2326 | 0.6844 | − 0.6689 | − 0.1480 | − 0.5726 |
| | (0.0040) | (0.0112) | (0.0000) | (0.0890) | (0.0000) |
| $\beta_4$ | 3.2917 | 9.1242 | 6.5294 | 1.9581 | 4.3747 |
| | (0.0000) | (0.0007) | (0.0000) | (0.0174) | (0.0000) |
| Adjusted $R^2$ | 0.1889 | 0.1173 | 0.2239 | 0.1316 | 0.1746 |
| $F$ | 256.47 | 146.73 | 296.03 | 164.84 | 221.15 |
| *Puts* | | | | | |
| $\beta_0$ | − 0.0532 | 4.9358 | 0.2921 | 0.2239 | 0.2368 |
| | (0.7683) | (0.0000) | (0.1762) | (0.2441) | (0.2316) |
| $\beta_1$ | − 0.6068 | − 11.0882 | − 1.6823 | − 1.2536 | − 1.3584 |
| | (0.0731) | (0.0000) | (0.0000) | (0.0006) | (0.0003) |

Table 7 (*continued*)

| Coefficients | BS | AHBS | GARCH | VG | SV |
|---|---|---|---|---|---|
| Panel B: 1 week ahead out-of-sample pricing errors | | | | | |
| *Puts* | | | | | |
| $\beta_2$ | 0.7904 | 6.7608 | 1.4809 | 1.1080 | 1.1680 |
| | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.0000) |
| $\beta_3$ | − 0.1428 | 1.2465 | − 0.2326 | − 0.1572 | − 0.0678 |
| | (0.0608) | (0.0000) | (0.0156) | (0.0418) | (0.4437) |
| $\beta_4$ | 0.5634 | − 7.2371 | 1.9463 | 1.3210 | 1.7001 |
| | (0.4252) | (0.0292) | (0.0093) | (0.0562) | (0.0260) |
| Adjusted $R^2$ | 0.2473 | 0.1317 | 0.2931 | 0.2528 | 0.2339 |
| $F$ | 372.87 | 172.63 | 409.53 | 379.29 | 331.00 |

This table reports the results for regression on out-of-sample pricing errors. The regression below is based on the equation

$$\varepsilon_{n.t} = \beta_0 + \beta_1(S_t/K_n) + \beta_2(S_t/K_n)^2 + \beta_3\tau_n + \beta_4 r_t + \eta_n(t)$$

where $\varepsilon_{n.t}$ is the absolute percentage error of the option $n$ on day $t$; $S_t/K_n$, $\tau_n$ and $r_t$ respectively represent the moneyness, the time-to-maturity of the option contract and the risk-free interest rate at date $t$. The estimated regression coefficients are presented in this table together with their $p$-values, which are shown in parenthesis. Adjusted $R^2$ values and $F$-statistics for the linear regression model are reported in the last two rows of the table. BS denotes the Black and Scholes model, AHBS denotes the ad hoc Black and Scholes procedure that fits to the implied volatility surface, GARCH denotes Heston and Nandi's (2000) GARCH type discrete model, VG denotes Madan et al.'s (1998) variance gamma model, and SV denotes Heston's (1993) continuous-time stochastic volatility model.

underlying asset price volatility alone, and carry out a delta-neutral hedge, employing only the underlying asset as the hedging instrument.

We implement hedging with the following method. Consider hedging a short position in an option, $O(t, \tau; K)$ with $\tau$ periods to maturity and strike price of $K$. Let $\Delta_S(t)$ be the number of shares of the underlying asset to be purchased, and $\Delta_0(=O(t, \tau; K) - \Delta_S(t)S_t)$ be the residual cash positions. We consider the delta hedging strategy of $\Delta_S = \partial O(t, \tau; K)/\partial S_t)$ and $\Delta_0(t)$. The delta, for a put option, is negative, which means that a short position in put options should be hedged with a short position in the underlying stock.

To examine the hedging performance, we use the following steps. First, on day $t$, we short an option, and construct a hedging portfolio by buying $\Delta_S(t)$ shares of the underlying asset,[8] and investing $\Delta_0(t)$ in a risk-free bond. To compute $\Delta_S(t)$, we use estimated structural parameters from the previous trading day and the current day's asset price. Second, we liquidate the position after the next trading day or the next week. Then we compute the hedging error as the difference between the value of the replicating portfolio and the option price at the time of liquidation:

$$\varepsilon_t = \Delta_S S_{t+\Delta t} + \Delta_0 e^{r\Delta t} - O(t+\Delta t, \tau - \Delta t; K). \tag{17}$$

Tables 8 and 9 present 1 day and 1 week hedging errors over alternative moneyness categories, respectively. SV has the best hedging performance irrespective of the

[8] In case of put options, some shares of the underlying asset are shorted because $\Delta_s(t)$ is negative.

rebalancing period. Exceptionally, for 1 week ahead hedging errors of put options, BS shows the best performance. But the difference among models is not so large. GARCH shows similar results with other models contrary to the case of pricing. We recognize that GARCH shows a weak point in pricing but a strong point in hedging, i.e. forecasting volatilities. In each moneyness category, the hedging errors are highest for OTM options and get smaller as we move to ITM options. This pattern is true for every model and for each rebalancing frequency.

All models show positive mean hedging errors on average. This can be interpreted with the negative risk premium associated with the volatility risk (Bakshi and Kapadia, 2001). Purchased options are hedged against significant market declines. The reason is that increased realized volatility coincides with downward market moves. One economic interpretation is that buyers of market volatility are willing to pay a premium for downside protection. In our hedging implementation, the replicating portfolio holds a short position in volatility and the effect of change in volatility is not taken into account. So, the replicating portfolio (= delta-hedged portfolio) has positive gains because of a negative risk premium of the volatility risk.

## 6. Conclusion

We have studied pricing and hedging performances of alternative stochastic volatility option pricing models: Black and Scholes (1973) model, the ad hoc Black and Scholes procedure that fits to the implied volatility surface, Heston and Nandi's (2000) GARCH type discrete model, Madan et al.'s (1998) variance gamma model and Heston's (1993) continuous-time stochastic volatility model. We estimate each model from the daily cross-section of the KOSPI 200 index option prices. Our results are as follows.

First, SV outperforms other models in terms of in-sample pricing, out-of-sample pricing and hedging performances. Second, the addition of the stochastic volatility term does not resolve the "volatility smiles" effects, but it reduces the effects. Third, BS is competent in option pricing with the advantage of simplicity. This result reflects the actuality that most market practitioners in the emerging markets still use BS. Finally,

Notes to Table 8:
This table reports 1 day ahead hedging error for the KOSPI 200 option with respect to moneyness. Only the underlying asset is used as the hedging instrument. Parameters and spot volatility implied by all options of the previous day are used to establish the current day's hedge portfolio, which is then liquidated the following day. For each option, its hedging error is the difference between the replicating portfolio value and its market price. Denoting $\varepsilon_n$ hedging errors, hedging performance is evaluated by: (1) mean percentage error (MPE), $(\sum_{n=1}^{N}\varepsilon_n/O_n)/N$, (2) mean absolute percentage error (MAPE), $(\sum_{n=1}^{N}|\varepsilon_n|/O_n)/N$, (3) mean absolute error (MAE), $(\sum_{n=1}^{N}|\varepsilon_n|)/N$, and (4) mean squared error (MSE), $(\sum_{n=1}^{N}(\varepsilon_n)^2)/N$, where $N$ is the total number of options in a particular moneyness category. BS denotes the Black and Scholes model, AHBS denotes the ad hoc Black and Scholes procedure that fits to the implied volatility surface, GARCH denotes Heston and Nandi's (2000) GARCH type discrete model, VG denotes Madan et al.'s (1998) variance gamma model, and SV denotes Heston's (1993) continuous-time stochastic volatility model.

GARCH is the worst performer. In the emerging markets such as the Korean KOSPI 200 index options market with liquidity concentrated in deep OTM options, GARCH does not perform appropriately.

Table 8
One day ahead hedging errors

| | *S*/*K* | < 0.94 | 0.94 – 0.97 | 0.97 – 1.00 | 1.00 – 1.03 | 1.03 – 1.06 | ≥ 1.06 | All |
|---|---|---|---|---|---|---|---|---|
| *Panel A: Calls* | | | | | | | | |
| MPE | BS | 0.1924 | 0.2627 | 0.1429 | 0.0765 | 0.0348 | 0.0049 | 0.1449 |
| | AHBS | 0.1924 | 0.2652 | 0.1446 | 0.0773 | 0.0356 | 0.0044 | 0.1456 |
| | GARCH | 0.2645 | 0.2259 | 0.1027 | 0.0583 | 0.0325 | 0.0148 | 0.1559 |
| | VG | 0.1973 | 0.2672 | 0.1345 | 0.0700 | 0.0314 | 0.0052 | 0.1447 |
| | SV | 0.1630 | 0.2516 | 0.1289 | 0.0730 | 0.0335 | 0.0040 | 0.1297 |
| MAPE | BS | 0.4152 | 0.3713 | 0.2272 | 0.1390 | 0.0971 | 0.0612 | 0.2727 |
| | AHBS | 0.4232 | 0.3740 | 0.2286 | 0.1397 | 0.0981 | 0.0615 | 0.2764 |
| | GARCH | 0.4147 | 0.3730 | 0.2301 | 0.1477 | 0.0992 | 0.0620 | 0.2747 |
| | VG | 0.4001 | 0.3749 | 0.2247 | 0.1394 | 0.0975 | 0.0606 | 0.2676 |
| | SV | 0.3941 | 0.3628 | 0.2141 | 0.1372 | 0.0967 | 0.0612 | 0.2611 |
| MAE | BS | 0.4634 | 0.6377 | 0.6299 | 0.5774 | 0.5728 | 0.5833 | 0.5561 |
| | AHBS | 0.4701 | 0.6426 | 0.6336 | 0.5811 | 0.5791 | 0.5874 | 0.5614 |
| | GARCH | 0.4644 | 0.6581 | 0.6576 | 0.6241 | 0.5899 | 0.4644 | 0.5729 |
| | VG | 0.4467 | 0.6440 | 0.6263 | 0.5807 | 0.5764 | 0.5782 | 0.5511 |
| | SV | 0.4367 | 0.6220 | 0.5904 | 0.5725 | 0.5717 | 0.5860 | 0.5380 |
| MSE | BS | 0.6890 | 1.3525 | 1.2467 | 0.9390 | 0.6737 | 0.7940 | 0.9260 |
| | AHBS | 0.6968 | 1.3618 | 1.2521 | 0.9448 | 0.6930 | 0.7967 | 0.9340 |
| | GARCH | 0.6618 | 1.3569 | 1.2475 | 0.9885 | 0.7158 | 0.8016 | 0.9289 |
| | VG | 0.6635 | 1.3746 | 1.2232 | 0.9366 | 0.6753 | 0.7973 | 0.9173 |
| | SV | 0.6073 | 1.2730 | 1.0578 | 0.9209 | 0.6723 | 0.8077 | 0.8547 |
| *Panel B: Puts* | | | | | | | | |
| MPE | BS | 0.0011 | 0.0310 | 0.0588 | 0.1347 | 0.2256 | 0.1707 | 0.1132 |
| | AHBS | 0.0016 | 0.0314 | 0.0599 | 0.1375 | 0.2293 | 0.1696 | 0.1140 |
| | GARCH | 0.0082 | 0.0186 | 0.0243 | 0.0950 | 0.2041 | 0.2164 | 0.1131 |
| | VG | 0.0042 | 0.0326 | 0.0538 | 0.1180 | 0.2010 | 0.1645 | 0.1055 |
| | SV | 0.0016 | 0.0266 | 0.0554 | 0.1264 | 0.2117 | 0.1569 | 0.1048 |
| MAPE | BS | 0.0652 | 0.1005 | 0.1376 | 0.2358 | 0.3539 | 0.3202 | 0.2197 |
| | AHBS | 0.0657 | 0.1005 | 0.1375 | 0.2364 | 0.3558 | 0.3294 | 0.2229 |
| | GARCH | 0.0639 | 0.1043 | 0.1464 | 0.2473 | 0.3617 | 0.3467 | 0.2319 |
| | VG | 0.0641 | 0.1007 | 0.1425 | 0.2428 | 0.3518 | 0.3100 | 0.2177 |
| | SV | 0.0653 | 0.0973 | 0.1347 | 0.2328 | 0.3513 | 0.3107 | 0.2149 |
| MAE | BS | 0.7003 | 0.5997 | 0.5546 | 0.5521 | 0.4744 | 0.3402 | 0.5063 |
| | AHBS | 0.7039 | 0.5997 | 0.5547 | 0.5534 | 0.4760 | 0.3482 | 0.5097 |
| | GARCH | 0.6748 | 0.6262 | 0.5952 | 0.5908 | 0.5024 | 0.3780 | 0.5323 |
| | VG | 0.6827 | 0.5993 | 0.5742 | 0.5663 | 0.4820 | 0.3303 | 0.5067 |
| | SV | 0.6982 | 0.5831 | 0.5447 | 0.5454 | 0.4684 | 0.3306 | 0.4984 |
| MSE | BS | 0.9600 | 0.7854 | 0.6520 | 0.7861 | 0.5939 | 0.4442 | 0.6633 |
| | AHBS | 0.9731 | 0.7896 | 0.6539 | 0.7882 | 0.5965 | 0.4512 | 0.6688 |
| | GARCH | 0.9031 | 0.8359 | 0.7318 | 0.8454 | 0.6287 | 0.4809 | 0.6968 |
| | VG | 0.9081 | 0.7960 | 0.6795 | 0.8037 | 0.5976 | 0.4276 | 0.6590 |
| | SV | 0.9442 | 0.7210 | 0.6281 | 0.7752 | 0.5683 | 0.4292 | 0.6407 |

Summarizing all findings, SV is found to be a recommended option pricing model. Also, BS can be a competitive model in the emerging markets like the Korean KOSPI 200 index option market. Also, our results suggest possible avenues for future investigation. The remaining and persistent pricing errors and hedging errors based on SV suggest that a

Table 9
One week ahead hedging errors

| | *S/K* | < 0.94 | 0.94 – 0.97 | 0.97 – 1.00 | 1.00 – 1.03 | 1.03 – 1.06 | ≥ 1.06 | All |
|---|---|---|---|---|---|---|---|---|
| *Panel A: Calls* | | | | | | | | |
| MPE | BS | − 0.0237 | 0.2892 | 0.2210 | 0.1484 | 0.0617 | 0.0013 | 0.0960 |
| | AHBS | − 0.0032 | 0.3076 | 0.2325 | 0.1551 | 0.0659 | 0.0017 | 0.1091 |
| | GARCH | 0.1422 | 0.1455 | 0.1106 | 0.0850 | 0.0394 | 0.0183 | 0.1062 |
| | VG | 0.0132 | 0.2660 | 0.2031 | 0.1332 | 0.0481 | − 0.0028 | 0.0990 |
| | SV | − 0.0683 | 0.2745 | 0.2157 | 0.1486 | 0.0607 | − 0.0054 | 0.0775 |
| MAPE | BS | 0.8632 | 0.5129 | 0.3190 | 0.2046 | 0.1351 | 0.0927 | 0.4814 |
| | AHBS | 0.8760 | 0.5252 | 0.3255 | 0.2078 | 0.1339 | 0.0931 | 0.4891 |
| | GARCH | 0.8838 | 0.5239 | 0.3191 | 0.2174 | 0.1563 | 0.1019 | 0.4949 |
| | VG | 0.8509 | 0.5306 | 0.3198 | 0.2038 | 0.1370 | 0.0897 | 0.4796 |
| | SV | 0.8570 | 0.5083 | 0.3177 | 0.2053 | 0.1349 | 0.0892 | 0.4759 |
| MAE | BS | 0.9011 | 0.8887 | 0.8560 | 0.8579 | 0.8165 | 0.9063 | 0.8803 |
| | AHBS | 0.9197 | 0.9087 | 0.8708 | 0.8694 | 0.8069 | 0.9091 | 0.8933 |
| | GARCH | 0.9511 | 0.9729 | 0.9149 | 0.9432 | 0.9695 | 0.9863 | 0.9541 |
| | VG | 0.9010 | 0.9281 | 0.8623 | 0.8621 | 0.8399 | 0.8882 | 0.8877 |
| | SV | 0.9002 | 0.8923 | 0.8534 | 0.8567 | 0.8170 | 0.8811 | 0.8766 |
| MSE | BS | 1.8109 | 1.5546 | 1.3508 | 1.3062 | 1.1976 | 1.6873 | 1.5684 |
| | AHBS | 1.8341 | 1.5868 | 1.3875 | 1.3474 | 1.1354 | 1.6636 | 1.5841 |
| | GARCH | 1.9717 | 1.7930 | 1.5503 | 1.5597 | 1.8224 | 1.9623 | 1.8131 |
| | VG | 1.8203 | 1.6599 | 1.3568 | 1.3029 | 1.2650 | 1.5354 | 1.5753 |
| | SV | 1.8027 | 1.5370 | 1.3087 | 1.3069 | 1.1956 | 1.4169 | 1.5198 |
| *Panel B: Puts* | | | | | | | | |
| MPE | BS | − 0.0326 | 0.0491 | 0.1179 | 0.2168 | 0.3145 | 0.2345 | 0.1527 |
| | AHBS | − 0.0312 | 0.0514 | 0.1255 | 0.2262 | 0.3280 | 0.2399 | 0.1588 |
| | GARCH | − 0.0394 | − 0.0117 | 0.0130 | 0.0760 | 0.1638 | 0.3273 | 0.1145 |
| | VG | − 0.0310 | 0.0458 | 0.1030 | 0.1859 | 0.2528 | 0.2210 | 0.1340 |
| | SV | − 0.0359 | 0.0470 | 0.1163 | 0.2103 | 0.2900 | 0.2182 | 0.1425 |
| MAPE | BS | 0.1172 | 0.1637 | 0.2206 | 0.3480 | 0.5503 | 0.6274 | 0.3663 |
| | AHBS | 0.1176 | 0.1644 | 0.2214 | 0.3458 | 0.5527 | 0.6499 | 0.3728 |
| | GARCH | 0.1197 | 0.1728 | 0.2273 | 0.3637 | 0.5664 | 0.6336 | 0.3749 |
| | VG | 0.1174 | 0.1709 | 0.2264 | 0.3547 | 0.5716 | 0.6271 | 0.3710 |
| | SV | 0.1189 | 0.1640 | 0.2230 | 0.3488 | 0.5489 | 0.6098 | 0.3609 |
| MAE | BS | 1.3115 | 0.9688 | 0.8687 | 0.8504 | 0.8094 | 0.6309 | 0.8885 |
| | AHBS | 1.3138 | 0.9726 | 0.8691 | 0.8417 | 0.8083 | 0.6482 | 0.8928 |
| | GARCH | 1.2992 | 1.0494 | 0.9222 | 0.9262 | 0.8660 | 0.6549 | 0.9287 |
| | VG | 1.3023 | 1.0114 | 0.8913 | 0.8635 | 0.8253 | 0.6307 | 0.9001 |
| | SV | 1.3338 | 0.9693 | 0.8810 | 0.8502 | 0.8048 | 0.6137 | 0.8902 |
| MSE | BS | 3.4001 | 1.5722 | 1.3189 | 1.3674 | 1.2784 | 0.8106 | 1.6004 |
| | AHBS | 3.3760 | 1.5780 | 1.3182 | 1.3557 | 1.2711 | 0.8328 | 1.6001 |
| | GARCH | 3.7834 | 1.9992 | 1.4874 | 1.5479 | 1.4926 | 0.8880 | 1.8248 |
| | VG | 3.4187 | 1.7246 | 1.3669 | 1.3965 | 1.3045 | 0.8265 | 1.6445 |
| | SV | 3.5695 | 1.5894 | 1.3339 | 1.3314 | 1.2588 | 0.7766 | 1.6226 |

jump component may further improve performances.[9] One reason is that in emerging markets a jump risk is probably more severe than in developed markets. Therefore modeling a jump risk in option pricing may be more relevant for emerging markets.

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## Appendix A

The purpose of this appendix is to display the risk neutral probability of each model.

(1) GARCH

$$P_1 = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{e^{-i\phi \ln[K]} f(\phi - i)}{i\phi}\right] d\phi,$$

$$P_2 = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{e^{-i\phi \ln[K]} f(\phi)}{i\phi}\right] d\phi,$$

where $f(\phi) = \exp(A(t;\ T,\ \phi) + B(t;\ T,\ \phi)h(t+1) + i\phi\ln[S_t])$; $A(t;\ \tau,\ \phi)$ and $B(t;\tau,\phi)$ are computed recursively as

$$\begin{aligned} A(t;\tau,\phi) &= A(t+1;\tau-1,\phi) + i\phi r + B(t+1;\tau-1,\phi) \\ &\quad - \frac{1}{2}\ln[1 - 2\alpha B(t+1;\tau-1,\phi)], \end{aligned}$$

Notes to Table 9:

This table reports 1 week ahead hedging error for the KOSPI 200 option with respect to moneyness. Only the underlying asset is used as the hedging instrument. Parameters and spot volatility implied by all options of the previous day are used to establish the current day's hedge portfolio, which is then liquidated the next week. For each option, its hedging error is the difference between the replicating portfolio value and its market price. Denoting $\varepsilon_n$ hedging errors, hedging performance is evaluated by: (1) mean percentage error (MPE), $(\sum_{n=1}^{N} \varepsilon_n / O_n)/N$, (2) mean absolute percentage error (MAPE), $(\sum_{n=1}^{N} |\varepsilon_n| / O_n)/N$, (3) mean absolute error (MAE), $(\sum_{n=1}^{N} |\varepsilon_n|)/N$, (4) and mean squared error (MSE), $(\sum_{n=1}^{N} (\varepsilon_n)^2)/N$, where $N$ is the total number of options in a particular moneyness category. BS denotes the Black and Scholes model, AHBS denotes the ad hoc Black and Scholes procedure that fits to the implied volatility surface, GARCH denotes Heston and Nandi's (2000) GARCH type discrete model, VG denotes Madan et al.'s (1998) variance gamma model, and SV denotes Heston's (1993) continuous-time stochastic volatility model.

[9] Bakshi et al. (1997) also mentioned, "The fact that such jumps and crashes are allowed to be discontinuous over time makes these models more flexible than the diffusion–stochastic–volatility model, in internalizing the desired return distributions, especially at short time horizons."

$$B(t;\tau,\phi) = i\phi\left(\gamma - \frac{1}{2}\right) - \frac{1}{2}\gamma^2 + \beta B(t+1;\tau-1,\phi) + \frac{(i\phi-\gamma)^2/2}{1-2\alpha\beta(t+1;\tau-1,\phi)}, \quad 9$$

with terminal conditions ($A(T;\,0,\,\phi)+B(T;\,0,\,\phi)=0$.

(2) VG

$$P_1 = \varphi\left[d\sqrt{\frac{1-c_1}{v}}, (\alpha+\sigma)\sqrt{\frac{v}{1-c_1}}, \frac{\tau}{v}\right],$$

$$P_2 = \varphi\left[d\sqrt{\frac{1-c_2}{v}}, \alpha\sqrt{\frac{v}{1-c_2}}, \frac{\tau}{v}\right],$$

where

$$d = \frac{1}{\sigma}\left[\ln\left(\frac{S_t}{K}\right) + r\tau + \frac{\tau}{v}\ln\left(\frac{1-c_1}{1-c_2}\right)\right],$$

$$c_1 = \frac{v(\alpha+\sigma)^2}{2},\ \ c_2 = \frac{v\alpha^2}{2},$$

$$\varphi(a,b,\gamma) = \int_0^\infty \Phi\left(\frac{a}{\sqrt{g}} + b\sqrt{g}\right)\frac{g^{\gamma-1}\mathrm{e}^{-g}}{\Gamma(\gamma)}\,\mathrm{d}g.$$

$\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution, and $\Gamma(\cdot)$ is the gamma function.

(3) SV

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{\mathrm{e}^{-i\phi\ln[K]}f_j(x,v_t,\tau;\phi)}{i\phi}\right]\mathrm{d}\phi \quad (j=1,2)$$

where

$$f_j(x,v,\tau,\phi) = \exp[C(\tau;\phi) + D(\tau;\phi)v_t + i\phi x],$$

$$C(\tau;\phi) = r\phi i\tau + \frac{a}{\sigma^2}\left\{(b_j - \rho\sigma\phi i + d)\tau - 2\ln\left[\frac{1-ge^{\mathrm{d}\tau}}{1-g}\right]\right\},$$

$$D(\tau;\phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}\left[\frac{1-e^{d\tau}}{1-ge^{d\tau}}\right],$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d},\ d = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2\mu_j\phi i - \phi^2)},$$

$$a = \kappa\theta,\ b_1 = \kappa - \rho\sigma,\ b_2 = \kappa,\ \mu_1 = 1/2,\ \mu_2 = -1/2.$$

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