IS IT IMPORTANT TO CONSIDER THE JUMP COMPONENT FOR PRICING AND HEDGING SHORT-TERM OPTIONS?

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The usefulness of the jump component for pricing and hedging short-term options is studied for the KOSPI (Korean Composite Stock Price Index) 200 Index options. It is found that jumps have only a marginal effect and stochastic volatility is of the most importance. There is evidence of jumps in the underlying index but no evidence of jumps in the corresponding index options. However, these results may not be valid for individual equity options. © 2005 Wiley Periodicals, Inc. Jrl Fut Mark 25:989–1009, 2005

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INTRODUCTION

It has long been asserted that jumps or stochastic volatility may provide additional flexibility in capturing the salient features of equity returns, including skewness and leptokurtosis.[1] In a recent study, Andersen, Benzoni, and Lund (2002) show that discrete jump component and stochastic volatility with a negative correlation between asset return and volatility are significant in the S&P 500 market. In studies of the S&P 500 options market, Bakshi, Cao, and Chen (1997) find it important to incorporate stochastic volatility for pricing and preserving internal consistency. It is found that the stochastic volatility is the most important factor, and jumps show only marginal effects; these results differ from those of the underlying index market. Furthermore, Bakshi, Cao, and Chen (2000) find that it is of first-order importance even for the long-term option such as LEAPS (long-term equity anticipation securities) to take stochastic volatility into consideration. In short, the stochastic volatility is the most significant factor regardless of the maturity of the options.

If so, one may wonder whether it is completely unnecessary to consider jumps for pricing and hedging options. It is a common understanding in the literature that jumps are important for pricing and hedging short-term options.[2] Moreover, if volatility follows a pure diffusion, the implied path of continuous sample may be incapable of generating a sufficiently volatile return distribution over short horizons to justify the observed prices of derivatives instruments.

The objective of this article is to examine the importance of the jump component for pricing and hedging short-term options. Previous studies have tested the usefulness of jumps in the options market; however, these have not separated the cases where only jumps themselves exist from those cases where jumps are additionally added to stochastic volatility. For instance, Merton (1976), Bates (1991), and Jorion (1988) have tested the absolute effect of jumps by comparing the jump-diffusion option pricing model (henceforth the J model) to the Black and Scholes (1973) model (henceforth the BS model). Also, Bakshi et al. (1997) have examined the marginal effect of the J model by comparing

[1]See, for example, Jorion (1988), Kim, Oh, and Brooks (1994), Eraker, Johannes, and Polson (2000).

[2]Several studies have noted that the incorporation of a jump component is essential when pricing options that are close to maturity. Bakshi, Cao, and Chen (1997) find the significance of jumps for pricing short-term options. Also, a number of other articles illustrate the importance of jumps for pricing and hedging options—see Ball and Torous (1985), Jorion (1988), Naik and Lee (1990), Naik (1993), and Das and Sundaram (1999).

the stochastic-volatility model (henceforth the SV model) and stochastic-volatility jump-diffusion model (henceforth the SVJ model). This article examines both the absolute and marginal effects of the jump component, which is well known to play an important role for the short-term options. The subject of study is the Korean Composite Stock Price Index (henceforth KOSPI) 200 options market, the largest options market in the world despite its short history.[3] Focusing on this market will also be useful, because there is an excellent liquidity in the near contract. In the Korean stock index market, jumps and stochastic volatility are found under physical measure. Chang (1997) and Kim and Chang (1996) test the existence of jump component by using the model to admit both the transient jump and the conditional volatility of KOSPI 200 and KOSPI, respectively. They find that after taking the heteroscedasticity effect into account, there still exist significant jump components in well-diversified portfolios such as KOSPI 200 and KOSPI. Chang (2003) estimates the continuous-time stochastic-volatility diffusion model by using the EMM of Gallant and Tauchen (1997) and finds that the stochastic-volatility component is a significant factor in the KOSPI market. Similar to cases in the S&P 500 index market, stochastic volatility and jumps are important factors for describing the underlying index. This study assesses the usefulness of the jump component for pricing and hedging short-term options.

The J model of Bates (1991) is compared with the SV and the SVJ models of Bakshi et al. (1997) from three perspectives: in-sample pricing, out-of-sample pricing, and hedging performances. If the SVJ model turns out to be the best, and the J model the worst, and the differences between the SV and the SVJ models not much, it will imply that the jump component has only marginal effects even for short-term options. In contrast, if the SVJ model is the best, the SV model the worst, and the difference between the SVJ and the J models not much, the jump component will be the most important factor for short-term options in an absolute sense.

The outline of this article is as follows. Alternative models are reviewed in the following section and the Estimation Procedure section describes the estimation method. Next, the data used for the analysis are described, and the Empirical Findings section describes the pricing and hedging performance of each model. Finally, the results are summarized.

[3]During the 5-year period from 1999 to 2003, in terms of trading volume, the KOSPI 200 options market has ranked first in the world. The Asian Risk magazine named the KOSPI 200 index options market as the derivatives market of the year 2001.

MODELS

SVJ Model

Bakshi et al. (1997) derived a closed-form option pricing model that incorporates stochastic volatility and random jumps. Under the risk-neutral measure, the underlying nondividend-paying stock price $S(t)$ and its components for any time t are given by the following:

$$\frac{dS(t)}{S(t)} = [R(t) - \lambda\mu_J]\,dt + \sqrt{V(t)}\,dZ_S(t) + J(t)\,dq(t) \tag{1}$$

$$dV(t) = [\theta_\nu - K_\nu V(t)]\,dt + \sigma_\nu\sqrt{V(t)}\,dZ_\nu(t) \tag{2}$$

$$\ln[1 + J(t)] \sim N(\ln[1 + \mu_J] - 1/2\sigma_J^2, \sigma_J^2) \tag{3}$$

where $R(t)$ is the instantaneous spot interest rate at time t, λ is the frequency of jumps per year, and $V(t)$ is the diffusion component of return variance (conditional on no jump occurring). $Z_S(t)$ and $Z_\nu(t)$ are standard Brownian motions, with $Cov_t[dZ_S(t), dZ_\nu(t)] = \rho\,dt$. $J(t)$ is the percentage jump size (conditional on a jump occurring) that is lognormally, identically, and independently distributed over time, with unconditional mean of μ_J. σ_J is the standard deviation of $\ln[1 + J(t)]$. $q(t)$ is a Poisson jump counter with intensity λ; that is, $\Pr[dq(t) = 1] = \lambda\,dt$ and $\Pr[dq(t) = 0] = 1 - \lambda\,dt$. κ_ν, θ_ν/κ_ν, and σ_ν are the speed of adjustment, long-run mean, and variation coefficient of the diffusion volatility $V(t)$, respectively. $q(t)$ and $J(t)$ are uncorrelated with each other or with $Z_S(t)$ and $Z_\nu(t)$.

For a European call option with strike price K and time to maturity τ, the closed form formula for price $C(t, \tau)$ at time t is as follows:

$$C(t, \tau) = S(t)P_1(t, \tau; S, R, V) - Ke^{-R\tau}P_2(t, \tau; S, R, V) \tag{4}$$

where the risk-neutral probabilities P_1 and P_2 are computed from inverting the respective characteristic functions of the following:

$$P_j(t, \tau; S(t), V(t)) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{\exp(-i\phi\ln K)f_j(t, \tau, S(t), V(t); \phi)}{i\phi}\right] \quad (j = 1, 2) \tag{5}$$

The characteristic functions, f_j's, are given in Equations (A-1) and (A-2) of the Appendix. The price of a European put on the same stock can be determined from the put–call parity. The option valuation model in Equation (5) contains the most existing models as special cases. For example, (i) the BS model is obtained by setting $\lambda = 0$ and $\theta_\nu = \kappa_\nu = \sigma_\nu = 0$;

and (ii) the SV model is obtained by setting $\lambda = 0$, where one may need to apply L'Hopital's rule to derive each special case from Equation (5). Also the constant-volatility jump-diffusion model of Bates (1991) is an embedded version of the SVJ model.

J Model

The specification used in Bates (1991) is now examined. The underlying asset with (possibly) asymmetric random jumps is described in Equation (1). The postulated process differs from the process in Merton (1976) and Ball and Torous (1983, 1985) in several significant aspects. First, jumps are allowed to be asymmetric, possibly with nonzero mean. For instance, values of the expected percentage jump size k greater (less) than zero imply that the distribution is positively (negatively) skewed relative to geometric Brownian motion. In addition, the jump risk is systematic and nondiversifiable.

As shown in the following formula, European call options are priced as the discounted expected value of their terminal payoffs, assuming that the terminal distribution is determined under the risk-neutral world:

$$C(t, \tau; S(t)) = e^{-r\tau} \sum_{n=0}^{\infty} \Pr^*(n \text{ jumps}) E_t^*[\max(S(t) - K, 0) \,|\, n \text{ jumps}]$$

$$= e^{-r\tau} \sum_{n=0}^{\infty} \left[e^{-\lambda^* \tau} (\lambda^* \tau)^n / n! \right] [S(t) e^{b(n)\tau} N(d_{1n}) - K N(d_{2n})] \quad (6)$$

where $b(n) = (r - \lambda^* \mu_J^*) + n(\ln \mu_J + 1)/\tau$,

$$d_{1n} = \frac{[\ln(S(t)/K) + b(n)\tau + \frac{1}{2}(V(t)\tau + n\sigma_J^2)]}{\sqrt{V(t)\tau + n\sigma_J^2}},$$

and $d_{2n} = d_{1n} - \sqrt{V(t)\tau + n\sigma_J^2}$.

ESTIMATION PROCEDURE

In implementing option pricing models, one always encounters the difficulty that the spot volatility and the structural parameters are unobservable. Because closed-form solutions are available for an option valuation, a natural candidate for the estimation of parameters in the pricing and hedging formula is a nonlinear least-squares procedure, involving a minimization of the sum of squared errors between the model and the market prices. As in Bakshi et al. (1997) and Bates (1995, 1996), the structural parameters are estimated together with the spot volatility of each model in the sample. Estimating parameters from the asset returns can be an alternative method, but historical data reflect only what has happened in the past. Furthermore, the procedure using historical data is not capable of identifying risk premiums, which must be estimated from the options data conditional on the estimates of other parameters. The important advantage of using option prices to estimate parameters is to allow one to use the forward-looking information contained in the option prices.

Let $O_i^*(t, \tau; K)$ denote the model price of option i on day t and $O_i(t, \tau; K)$ denote the market price of option i on day t. The parameters for calls and puts, respectively, are estimated.[4] That is, there are two sets of parameters for calls and puts day after day. To estimate parameters for each model, the sum of squared percentage errors between the model and the market prices are minimized as

$$\min_{\phi_t} \sum_{i=1}^{N} \left[\frac{O_i^*(t, \tau; K) - O_i(t, \tau; K)}{O_i(t, \tau; K)} \right]^2 \qquad (t = 1, \ldots, T) \tag{7}$$

where N denotes the number of options on day t, and T denotes the number of days in the sample. Conventionally, the objective function is used to minimize the sum of squared errors. However, the above function is adopted because the conventional method that gives more weight to relatively expensive in-the-money options makes a worse fit for out-of-the-money options.

DATA

Introduced in July 7 1997, the KOSPI 200 options are based on the KOSPI 200 index, consisting 200 constituent blue-chip stocks by Korea Stock Exchange (KSE). The KOSPI 200 options market started with an unprecedented enthusiasm. During the five years from 1999 to 2003, in terms of trading volume, the KSE options market was the most active options market in the world, with its annual trading volume reaching 1890 million contracts in 2002. Moreover, it is important to note that liquidity is concentrated in the nearest expiration contract. In other

[4]If both call and put option prices are used, in-the-money calls and out-of-the-money puts that are equivalent according to the put–call parity are used to estimate the parameters.

words, a tremendous trading volume is converged into one or two expiration contracts. Hence, it is ideal for the investigation of short-term options.

The sample period is from January 5, 2000 to July 31, 2002.[5] Minute-by-minute transaction prices for the KOSPI 200 options are obtained from the Korea Stock Exchange. The 91-day certificate-of-deposit (CD) yields are used as risk-free interest rates.[6] Because KOSPI 200 contracts are European style, index levels are adjusted for dividend payments before each option's expiration date. The KOSPI 200 index pays dividends quarterly at the end of March, June, September, and December, and these dates are used for adjustment dates.[7] The following rules are applied to filter the data needed for the empirical test.

1. Each day, only the last reported transaction price prior to 2:50 p.m.,[8] of each option contract is employed in the empirical test.[9]
2. As options with less than 6 days or more than 60 days to expiration may induce liquidity-related biases, they are excluded from the sample.
3. To mitigate the impact of price discreteness in option valuation, prices lower than 0.2 are eliminated.
4. Prices not satisfying option boundary conditions are excluded:

$$C_{t,\tau} \geq S_t - \sum_{s=1}^{\tau} e^{-r_{t,s}s} D_{t+s} - KB_{t,\tau}$$

$$P_{t,\tau} \geq KB_{t,\tau} - S_t + \sum_{s=1} e^{-r_{t,s}s} D_{t+s}$$

where $B_{t,\tau}$ is a zero-coupon bond that pays 1 in τ periods from time t, D_t is daily dividend at time t, and $r_{t,s}$ is the risk-free interest rate with maturity s at time t.

Option data are divided into several categories according to the moneyness S/K. Table I describes sample properties. Note that there are

[5]The sample includes the period of the 9/11 terrorism in 2001, and this shock may influence the results of the article differently. To validate this conjecture, when the period of the 9/11 terrorism was excluded from the sample, there was no change in the result.
[6]Because the Treasury-bill market in Korea is not liquid, the 91-day certificate-of-deposit (CD) yields are used for risk-free rates, in spite of the mismatch of maturity dates of options and interest rates.
[7]It is assumed that traders have perfect knowledge about future dividend payments because options in this study have short times to maturities.
[8]In the Korean stock market, there are simultaneous bids and offers from 2:50 p.m.
[9]Because the recorded KOPSI 200 index values are not equivalent to the daily closing index levels, there is no nonsynchronous price issue here, except that the KOSPI 200 index level itself may contain stale component stock prices at each point in time.

TABLE I
KOSPI 200 Options Data

S/K	*Calls*	*Puts*
<0.94	0.97 (4067)	12.83 (2885)
0.94–0.97	2.19 (1106)	6.04 (970)
0.97–1.00	3.11 (1021)	4.33 (975)
1.00–1.03	4.33 (941)	3.03 (986)
1.03–1.06	5.81 (808)	2.07 (945)
≥1.06	12.20 (1870)	0.87 (3628)
Subtotal	4.19 (9813)	5.31 (10389)

Note. This table reports average option price and the number of options, which are shown in parentheses, in terms of moneyness and type (call or put). The sample period is from January 5, 2000 to July 31, 2002. Daily information from the last transaction prices (prior to 2:50 p.m.) of each option contract is used to obtain the summary statistics. Moneyness of an option is defined as *S/K,* where *S* denotes the spot price and *K* denotes the strike price.

TABLE II
Implied Volatility

	S/K	*<0.94*	*0.94–0.97*	*0.97–1.00*	*1.00–1.03*	*1.03–1.06*	*≥1.06*
January 2000–	Calls	0.43	0.41	0.41	0.41	0.41	0.48
June 2000	Puts	0.58	0.42	0.42	0.42	0.42	0.46
July 2000–	Calls	0.51	0.48	0.47	0.47	0.47	0.50
December 2000	Puts	0.68	0.51	0.52	0.51	0.52	0.52
January 2001–	Calls	0.38	0.37	0.36	0.36	0.36	0.41
June 2001	Puts	0.42	0.39	0.38	0.38	0.38	0.41
July 2001–	Calls	0.34	0.31	0.30	0.29	0.28	0.40
December 2001	Puts	0.50	0.39	0.37	0.36	0.35	0.40
January 2002–	Calls	0.37	0.36	0.36	0.35	0.36	0.43
July 2002	Puts	0.43	0.39	0.38	0.37	0.37	0.39

Note. This table reports the implied volatilities calculated by inverting the Black-Scholes model separately for each moneyness category. The implied volatilities of individual options are then averaged within each moneyness category and across the days in the sample. Moneyness is defined as *S/K*, where *S* denotes the spot price and *K* denotes the strike price.

9,813 calls and 10,389 puts observed, with deep out-of-the-money options taking up to 41% of calls and 35% of puts. However, to remove such trading days that consist of the number of calls or puts less than eight, which is the number of parameters in the SVJ model, 6 and 3 days are excluded for calls and puts, respectively. Average numbers of daily options are 15.71 and 16.58, with minimums of 8 (and 8) and maximums of 33 (and 31) for call (and put) options, respectively. Table II presents the "volatility smile" effects for five consecutive subperiods. Six fixed intervals are employed for the degree of moneyness, and the mean of the implied volatility is computed over alternative subperiods. The Korean options market seems to be "smiling" or "sneering," independent of the subperiods employed in the estimation.

EMPIRICAL FINDINGS

In this section, empirical performances of each model are compared with respect to three metrics: (a) in-sample pricing performance, (b) out-of-sample pricing performance, and (c) hedging performance. The analysis is based on two measures: mean absolute errors (henceforth MAEs) and mean squared errors (henceforth MSEs). MAEs measure the magnitude of pricing errors, and MSEs measure the volatility of errors.

Estimated Parameters

Table III reports the daily averages and standard errors of daily spot volatility and relevant structural parameters. As shown in the table, those

TABLE III
Parameters

	Calls				*Puts*			
Parameters	*BS*	*J*	*SV*	*SVJ*	*BS*	*J*	*SV*	*SVJ*
λ		0.9084		1.2450		1.1627		1.4556
		(0.0664)		(0.0560)		(0.0722)		(0.0586)
μ_J		−0.0517		−0.0749		−0.0508		−0.0678
		(0.0082)		(0.0098)		(0.0112)		(0.0139)
σ_J		0.2869		0.1473		0.3281		0.1076
		(0.0124)		(0.0048)		(0.0136)		(0.0040)
κ_ν			6.8196	4.2780			7.2121	3.5853
			(0.2529)	(0.1910)			(0.2363)	(0.1505)
θ_ν			0.7493	0.1117			0.7475	0.1449
			(0.0344)	(0.0047)			(0.0364)	(0.0059)
σ_ν			1.0078	0.6297			1.0493	0.6405
			(0.0422)	(0.0244)			(0.0480)	(0.0244)
ρ			−0.1115	−0.2235			−0.1325	−0.2443
			(0.0185)	(0.0163)			(0.0179)	(0.0196)
$V(t)$	0.1803	0.3420	0.1775	0.1899	0.1849	0.3498	0.1805	0.1886
	(0.0025)	(0.0031)	(0.0037)	(0.0049)	(0.0037)	(0.0030)	(0.0042)	(0.0053)

Note. This table reports the structural parameters and the spot volatility of a given model, which are estimated by minimizing the sum of squared percentage pricing errors between the market price and the model price for each option. The daily average of the estimated parameters is reported first, followed by its standard error in parentheses. BS denotes the Black and Scholes (1973) model, and J denotes the Bates (1991) jump-diffusion model. SV and SVJ denote the Bakshi et al. (1997) stochastic-volatility model and stochastic-volatility jump-diffusion model, respectively.

estimated from calls and puts within each model are not much different. On the other hand, the parameters show large difference depending on the models. For instance, the frequency of the jump per year, λ, is larger for the SVJ model compared to that of the J model. Also, the standard deviation of the jump size, δ, is larger for the J model. For the stochastic-volatility model, both the speed of adjustment, κ_ν, and long-run mean, θ_ν/κ_ν, are larger for the SV model, compared to those for the SVJ model. The implied correlations of both models have negative values. The negative estimates indicate that the implied volatility and the index returns are negatively correlated and the implied distribution perceived by option traders is negatively skewed. Implied volatilities differ only by 15–20% per model, except for that of the J model, which has a value twice the others. Thus, implied volatilities show stability when compared to the structural parameters. However, note that the option prices and hedge ratios are extremely sensitive to the volatility input. In other words, even very minimal change in the volatility leads to substantially different results for pricing and hedging options.

In-Sample Pricing Performance

The in-sample pricing performance of each model is analyzed by comparing market prices with model prices computed by using the parameter estimates from the current day. Table IV reports in-sample valuation errors for the alternative models computed for all the options in the sample. First, with respect to all measures, the SVJ model shows the best performance, closely followed by the SV model. This is rather an obvious result when the use of larger number of parameters in the SVJ model is considered. Second, all the models show moneyness-based valuation errors. The models exhibit the worst fit for the out-of-the-money options. The fit, measured by MAEs, steadily improves as we move from out-of-the-money to in-the-money options. Overall, the SVJ model shows the best performance for in-sample pricing.

Now the SV and the SVJ models, the two best-performing models for in-sample pricing, are compared from a different perspective. The approach of Bates (1996, 2000) and Bakshi et al. (1997) is used, and a determination is made of whether each model's implied parameters are consistent with those implicit in the time series of the KOSPI 200 returns and the implied volatility. That is whether or not the daily averages

TABLE IV
In-Sample Pricing Errors

	S/K	*<0.94*	*0.94–0.97*	*0.97–1.00*	*1.00–1.03*	*1.03–1.06*	*≥1.06*	*All*
Panel A: Calls								
MAE	BS	0.16	0.22	0.26	0.31	0.34	0.31	0.24
	J	0.09	0.15	0.20	0.25	0.28	0.33	0.18
	SV	0.05	0.07	0.08	0.11	0.17	0.28	0.11
	SVJ	0.05	0.06	0.08	0.10	0.16	0.26	0.11
MSE	BS	0.04	0.07	0.08	0.08	0.14	0.74	0.19
	J	0.03	0.06	0.09	0.14	0.19	0.78	0.21
	SV	0.02	0.03	0.04	0.04	0.11	0.72	0.17
	SVJ	0.02	0.03	0.03	0.04	0.09	0.68	0.16
Panel B: Puts								
MAE	BS	0.40	0.32	0.26	0.21	0.18	0.15	0.26
	J	0.29	0.22	0.15	0.15	0.10	0.05	0.16
	SV	0.27	0.18	0.11	0.10	0.07	0.03	0.13
	SVJ	0.26	0.16	0.09	0.10	0.07	0.03	0.12
MSE	BS	0.34	0.16	0.06	0.11	0.05	0.04	0.15
	J	0.28	0.18	0.05	0.11	0.04	0.01	0.12
	SV	0.26	0.13	0.03	0.09	0.02	0.01	0.10
	SVJ	0.26	0.12	0.03	0.08	0.02	0.01	0.10

Note. This table reports in-sample pricing errors for the KOSPI 200 option with respect to moneyness. S/K is defined as moneyness, where S denotes the asset price and K denotes the strike price. Each model is estimated daily during the sample period and in-sample pricing errors are computed using estimated parameters from the current day. MAE denotes mean absolute errors and MSE denotes mean-squared errors. BS denotes the Black and Scholes (1973) model, and J denotes the Bates (1991) jump-diffusion model. SV and SVJ denote the Bakshi et al. (1997) stochastic-volatility model and stochastic-volatility jump-diffusion model, respectively.

of the implied parameters are similar in magnitude to those from the time-series counterparts is tested. Table V reports daily average of the implied parameter values from option prices and the maximum-likelihood (henceforth ML) estimates from the time series of the underlying asset. As is clearly shown in the table, the ML estimates of the parameters are different from their respective option-implied counterparts. Although it is not specifically stated here, the p values for the null hypothesis of equality between the ML estimates and the option implied parameters are not all in excess of 0.01%. The two models are misspecified and this is consistent with findings in Bates (1991) and Bakshi et al. (1997). Moreover, if the two models are compared, the option implied parameters of the SVJ model are more similar in order of magnitude to the ML estimates compared to SV model, except for the correlation

TABLE V
Consistency Tests

	Calls								*Puts*							
	SV				*SVJ*				*SV*				*SVJ*			
	κ_ν	θ_ν	σ_ν	ρ	κ_ν	θ_ν	σ_ν	ρ	κ_ν	θ_ν	σ_ν	ρ	κ_ν	θ_ν	σ_ν	ρ
Implied	6.8196	0.7493	1.0078	−0.1115	4.2780	0.1117	0.6297	−0.2235	7.2121	0.7475	1.0493	−0.1325	3.5853	0.1449	0.6405	−0.2443
MLE	0.2813	0.0478	0.2790		2.6397	0.4385	0.3026		0.1964	0.0366	0.2940		2.1835	0.3937	0.2714	
	(0.0135)	(0.0027)	(0.0091)		(0.1542)	(0.0126)	(0.0088)		(0.0112)	(0.0029)	(0.0189)		(0.1530)	(0.0233)	(0.0197)	
Time-series correlation				−0.1227				−0.0979				−0.1408				−0.0714

Note. This table reports daily average of the implied parameter values and the corresponding values of these under the true probability measure. The values under "Implied" correspond to the daily average of the respective implied parameter values. The corresponding values of these parameters under the physical measure are reported in the row under "MLE," and they are obtained by applying the maximum-likelihood estimation method to the spot variance time series. The row under "Time-series correlation" reports the sample time-series correlation between daily KOSPI 200 index returns and daily changes in the implied volatility of a given option pricing model. Standard errors are in parentheses. SV and SVJ denote the Bakshi et al. (1997) stochastic-volatility model and stochastic-volatility jump-diffusion model, respectively.

coefficient.[10] Thus, it may be concluded that the SVJ model, which shows the best performance for in-sample pricing is less misspecified than the SV model.

Out-of-Sample Pricing Performance

In-sample pricing performance can be biased because of the potential problem of overfitting to the data. In other words, a good in-sample fit might be a consequence of having a larger number of structural parameters. In the out-of-sample test, the presence of more parameters may actually cause overfitting and the model may suffer if the extra parameters do not improve its structural fitting. Out-of-sample pricing errors for the following day (week) are analyzed, with the use of the current day's estimated structural parameters and the spot volatility to price options for the following day (week).

Tables VI and VII, respectively, report 1-day- and 1-week-ahead out-of-sample valuation errors for alternative models computed over all the options in the sample. The SVJ model shows the best performance irrespective of measures and terms of out-of-sample pricing. With respect to moneyness-based errors, MAEs steadily decrease as one moves from deep out-of-the-money to in-the-money options for all the models. Generally, the SVJ model outperforms all the other models.

Second, pricing errors deteriorate as one moves from in-sample to out-of-sample pricing. The average of MAEs of all the models is 0.16 for the in-sample pricing, and grows to 0.23 for 1-day-ahead out-of-sample pricing. This difference is not much compared to 1-week-ahead out-of-sample pricing. One-week-ahead out-of-sample pricing errors grow to 0.31, almost twice as much as in-sample pricing errors. Although the SVJ model continues to outperform other models for out-of-sample pricing, the relative margin of performance is significantly less when compared to that of in-sample pricing case. The difference between errors of the BS and the SVJ models becomes smaller in the out-of-sample pricing. The ratio of the BS model to the SVJ model for MAEs is 2.18 for in-sample pricing errors. This ratio decreases to 1.30 and to 1.15 for 1-day-ahead and 1-week-ahead out-of-sample errors, respectively. In short, as the term of the out-of-sample pricing gets longer, the difference between errors of the two models becomes smaller. Also, the robust pricing performance of the SVJ model is not maintained as the term of out-of-sample pricing

[10]The differences between the option-implied correlation and the time-series correlation are smaller for the SV model than for the SVJ model.

TABLE VI
One-Day-Ahead Out-of-Sample Pricing Errors

	S/K	*<0.94*	*0.94–0.97*	*0.97–1.00*	*1.00–1.03*	*1.03–1.06*	*≥1.06*	*All*
Panel A: Calls								
MAE	BS	0.19	0.26	0.30	0.34	0.36	0.32	0.27
	J	0.14	0.23	0.27	0.30	0.33	0.36	0.24
	SV	0.13	0.20	0.22	0.24	0.27	0.32	0.20
	SVJ	0.13	0.19	0.21	0.23	0.27	0.32	0.20
MSE	BS	0.07	0.13	0.13	0.13	0.20	0.73	0.23
	J	0.06	0.14	0.15	0.18	0.25	0.76	0.24
	SV	0.06	0.13	0.11	0.12	0.20	0.76	0.22
	SVJ	0.05	0.12	0.11	0.12	0.20	0.76	0.22
Panel B: Puts								
MAE	BS	0.42	0.35	0.30	0.25	0.21	0.17	0.28
	J	0.36	0.31	0.25	0.23	0.19	0.10	0.23
	SV	0.36	0.30	0.25	0.23	0.18	0.10	0.22
	SVJ	0.35	0.29	0.25	0.22	0.18	0.10	0.22
MSE	BS	0.42	0.33	0.18	0.18	0.11	0.06	0.21
	J	0.42	0.32	0.17	0.17	0.11	0.06	0.20
	SV	0.40	0.28	0.16	0.16	0.10	0.05	0.19
	SVJ	0.36	0.25	0.14	0.16	0.09	0.05	0.18

Note. This table reports 1-day-ahead out-of-sample pricing errors for the KOSPI 200 option with respect to moneyness. S/K is defined as moneyness, where S denotes the asset price and K denotes the strike price. Each model is estimated every day during the sample period, and 1-day-ahead out-of-sample pricing errors are computed with estimated parameters from the previous trading day. MAE denotes mean absolute errors and MSE denotes mean-squared errors. BS denotes the Black and Scholes (1973) model, and J denotes the Bates (1991) jump-diffusion model. SV and SVJ denote the Bakshi, et al. (1997) stochastic-volatility model and stochastic-volatility jump-diffusion model, respectively.

increases, implying that the market consensus about the jump and volatility fear is volatile and that structural parameters must be changed frequently.

Finally, the impact of the jump component is examined. The MAEs of the J model are smaller compared to those of the BS model by 0.03 (0.02) and 0.05 (0.04) for 1-day- (1-week-) ahead pricing calls and puts, respectively. The SV model's MAEs decrease to 0.07 (0.03) and 0.06 (0.04) for 1-day- (1 week-) ahead pricing calls and puts, respectively, when compared to those of the BS model. In other words, the effects of the reduction of pricing errors for the SV model are much better compared to those for the J model. On the other hand, consider the SVJ model that adds the jump component to the SV model. The SVJ model reduces pricing errors of BS to 0.07 (0.04) and 0.06 (0.05) for 1-day- (1-week-) ahead pricing calls and puts, respectively. The difference between the performance of the SV model and the SVJ model is smaller than that between the performance of the J model and the SVJ model.

TABLE VII
One-Week-Ahead Out-of-Sample Pricing Errors

	S/K	*<0.94*	*0.94–0.97*	*0.97–1.00*	*1.00–1.03*	*1.03–1.06*	*≥1.06*	*All*
Panel A: Calls								
MAE	BS	0.28	0.34	0.37	0.40	0.40	0.33	0.33
	J	0.22	0.33	0.36	0.39	0.40	0.38	0.31
	SV	0.22	0.32	0.35	0.36	0.37	0.37	0.30
	SVJ	0.21	0.31	0.34	0.36	0.36	0.37	0.29
MSE	BS	0.11	0.21	0.25	0.25	0.31	0.84	0.31
	J	0.11	0.21	0.25	0.28	0.33	0.84	0.31
	SV	0.10	0.20	0.25	0.24	0.29	0.83	0.30
	SVJ	0.11	0.20	0.24	0.24	0.29	0.83	0.30
Panel B: Puts								
MAE	BS	0.46	0.41	0.37	0.32	0.28	0.23	0.34
	J	0.41	0.40	0.35	0.33	0.28	0.17	0.30
	SV	0.40	0.39	0.35	0.32	0.26	0.17	0.30
	SVJ	0.40	0.39	0.34	0.32	0.26	0.17	0.29
MSE	BS	0.51	0.46	0.30	0.33	0.19	0.11	0.31
	J	0.51	0.43	0.30	0.32	0.19	0.11	0.30
	SV	0.47	0.42	0.29	0.31	0.19	0.11	0.29
	SVJ	0.43	0.39	0.26	0.30	0.17	0.10	0.26

Note. This table reports 1-week-ahead out-of-sample pricing errors for the KOSPI 200 option with respect to moneyness. S/K is defined as moneyness, where S denotes the asset price and K denotes the strike price. Each model is estimated every day during the sample period, and 1-week-ahead out-of-sample pricing errors are computed with estimated parameters from 1-week ago. MAE denotes mean absolute errors and MSE denotes mean-squared errors. BS denotes the Black and Scholes (1973) model, and J denotes the Bates (1991) jump-diffusion model. SV and SVJ denote the Bakshi et al. (1997) stochastic-volatility model and stochastic-volatility jump-diffusion model, respectively.

To summarize, the jump component only has marginal effects, even for pricing short-term options. The SVJ model shows the best performance among all the models.

Hedging Performance

Hedging is often used as a tool of risk management to cover the positions in the underlying asset. Because there are several risk factors in the proposed models, the need for a perfect hedge may arise in situations where not only is the underlying price risk present, but so is volatility, or jump risk. To implement this hedging practice, it should be recognized that a perfect hedge is not practically feasible in the presence of stochastic jump sizes. In line with the measure of hedging performances in Dumas, Fleming, and Whaley (1998) and Gemmill and Saflekos (2000), the hedging error is defined as follows:

$$\varepsilon_t = \Delta O - \Delta O^* \tag{8}$$

TABLE VIII
One-Day-Ahead Hedging Errors

	S/K	*<0.94*	*0.94–0.97*	*0.97–1.00*	*1.00–1.03*	*1.03–1.06*	*≥1.06*	*All*
Panel A: Calls								
MAE	BS	0.16	0.25	0.26	0.30	0.33	0.36	0.24
	J	0.15	0.25	0.26	0.32	0.34	0.36	0.24
	SV	0.13	0.22	0.23	0.27	0.30	0.33	0.21
	SVJ	0.13	0.22	0.23	0.27	0.30	0.33	0.21
MSE	BS	0.07	0.17	0.14	0.21	0.31	0.70	0.22
	J	0.07	0.17	0.13	0.23	0.34	0.72	0.23
	SV	0.06	0.15	0.11	0.18	0.30	0.63	0.20
	SVJ	0.06	0.14	0.11	0.18	0.29	0.63	0.20
Panel B: Puts								
MAE	BS	0.38	0.36	0.29	0.27	0.22	0.12	0.25
	J	0.37	0.35	0.28	0.26	0.20	0.11	0.24
	SV	0.37	0.33	0.28	0.24	0.20	0.10	0.23
	SVJ	0.37	0.34	0.28	0.24	0.19	0.10	0.23
MSE	BS	0.40	0.42	0.19	0.20	0.12	0.04	0.20
	J	0.41	0.43	0.19	0.20	0.12	0.04	0.21
	SV	0.40	0.37	0.17	0.19	0.10	0.03	0.19
	SVJ	0.39	0.39	0.16	0.19	0.10	0.03	0.19

Note. This table reports 1-day-ahead hedging error for the KOSPI 200 option with respect to moneyness. Hedging errors are defined as the difference between the change in the reported market price and the change in the model's theoretical price from day t to day $t + 1$. MAE denotes mean absolute errors and MSE denotes mean-squared errors. BS denotes the Black and Scholes (1973) model, and J denotes the Bates (1991) jump-diffusion model. SV and SVJ denote Bakshi et al. (1997) stochastic-volatility model and stochastic-volatility jump-diffusion model, respectively.

where ΔO is the change in the reported market price from day t until day $t + 1$ or $t + 7$, and ΔO^* is the change in the theoretical price of the model.

Tables VIII and IX present 1-day and 1-week hedging errors over alternative moneyness categories, respectively. For both 1-day- and 1-week-ahead hedging activities, the SVJ model shows better performance with smaller errors, closely followed by the SV model. The tail-end models are the J model and the BS model for calls and puts, respectively. As discussed by Dumas et al. (1998) in a different context, the stability of the errors (or, at least, strongly and serially dependent as it suits a specification error) is an important factor to determine the ranking of the hedging performances. In the results of the out-of-sample pricing performance, MSEs of options are smaller in the SV or SVJ model compared to those in the BS or J model. In short, the hedging results are similar to the characteristics of MSEs of out-of-sample pricing, consistent with

TABLE IX
One-Week-Ahead Hedging Errors

	S/K	*<0.94*	*0.94–0.97*	*0.97–1.00*	*1.00–1.03*	*1.03–1.06*	*≥1.06*	*All*
Panel A: Calls								
MAE	BS	0.22	0.31	0.35	0.35	0.39	0.38	0.30
	J	0.22	0.32	0.37	0.37	0.41	0.41	0.31
	SV	0.20	0.30	0.36	0.36	0.37	0.36	0.29
	SVJ	0.20	0.30	0.35	0.34	0.37	0.37	0.29
MSE	BS	0.11	0.19	0.22	0.21	0.75	0.50	0.26
	J	0.12	0.20	0.25	0.27	0.87	0.56	0.29
	SV	0.10	0.17	0.24	0.23	0.66	0.48	0.24
	SVJ	0.10	0.17	0.22	0.21	0.66	0.49	0.24
Panel B: Puts								
MAE	BS	0.43	0.43	0.37	0.35	0.28	0.17	0.31
	J	0.43	0.42	0.37	0.34	0.28	0.16	0.31
	SV	0.42	0.42	0.37	0.34	0.28	0.16	0.30
	SVJ	0.42	0.43	0.36	0.34	0.27	0.15	0.30
MSE	BS	0.47	0.51	0.24	0.24	0.21	0.07	0.26
	J	0.47	0.54	0.28	0.25	0.23	0.07	0.27
	SV	0.47	0.48	0.27	0.26	0.25	0.07	0.27
	SVJ	0.45	0.46	0.25	0.23	0.23	0.06	0.25

Note. This table reports 1-week-ahead hedging error for the KOSPI 200 option with respect to moneyness. Hedging errors are defined as the difference between the change in the reported market price and the change in the model's theoretical price from day t to day $t + 7$. MAE denotes mean absolute errors and MSE denotes mean-squared errors. BS denotes the Black and Scholes (1973) model, and J denotes the Bates (1991) jump-diffusion model. SV and SVJ denote the Bakshi et al. (1997) stochastic-volatility model and stochastic-volatility jump-diffusion model, respectively.

the conjecture of Dumas et al. (1998). As a result, it is concluded that the stochastic-volatility term is the most important factor for hedging short-term options, same as in the pricing results.

Validation

Unike what is done when two measures (MAEs and MSEs) are used, here the models are compared by using the statistics to draw the concrete results.[11] Figure 1 summarizes the pairwise comparison results among the models by providing the t statistics of the probability that one model is better than the other. Regarding the out-of-sample pricing performance, the t statistics of the difference between each model's absolute pricing errors are shown in Panel A. The t statistics of the difference between each model's absolute hedging errors are shown in Panel B.

[11]Call and put options are included together in the analysis.

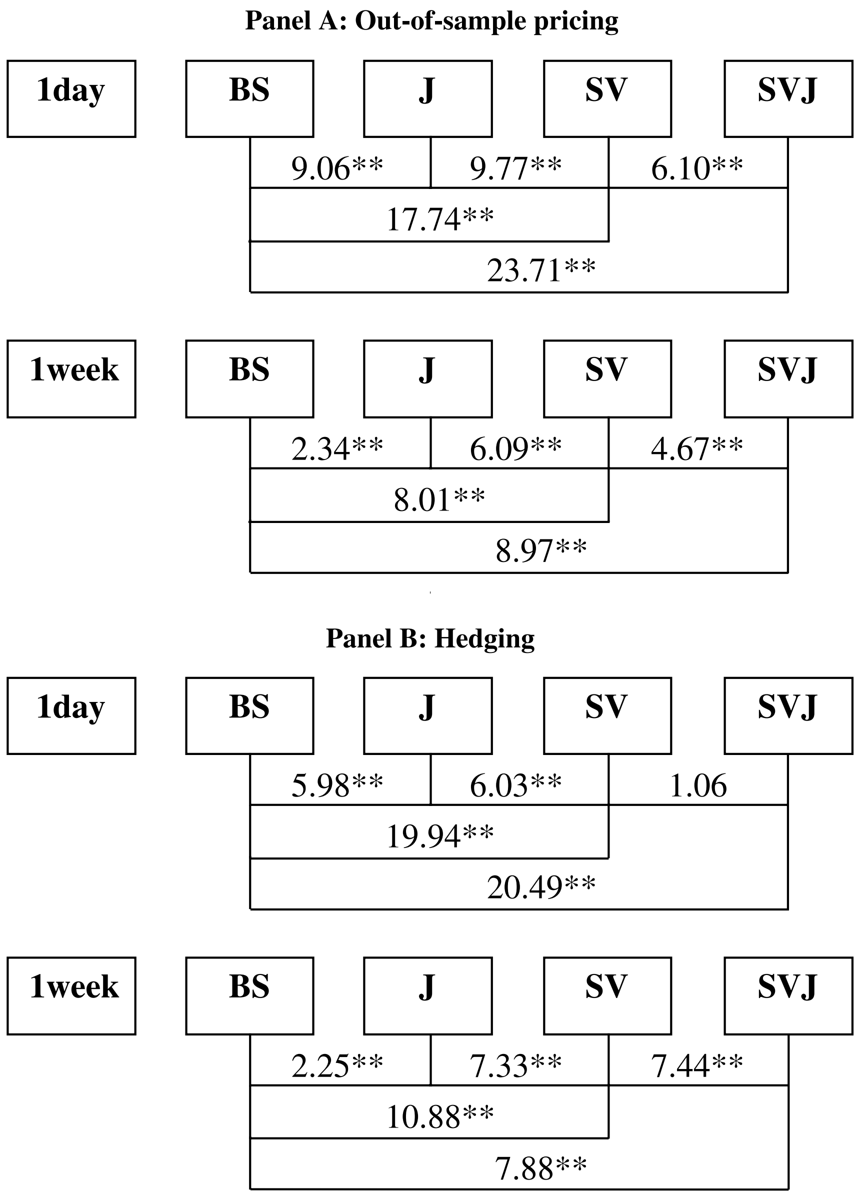

FIGURE 1
Differences among the errors of each model.

The comparison results are similar to those using MAEs. For both out-of-sample pricing and hedging performance, the SV model is exceedingly superior to the BS model. The SV model shows better performance than the J model. Also, the SVJ model outperforms the SV model. However, the significance of the superiority of the SVJ model compared to the BS model is similar to that of the SV model compared to the BS model. In the statistical analysis, the jump component shows minor marginal effects. Overall, the stochastic-volatility term is the most important factor for pricing and hedging short-term options.

CONCLUSION

It is known that the existence of jumps is significant under the dynamics of underlying assets only and not in the options market. Nonetheless, it has been widely documented in the literature that jumps may be important for pricing and hedging short-term options. Previous research has tested the usefulness of jumps, but have not separated the cases where only jumps themselves exist from those where jumps are additionally added to stochastic volatility. Here the impact of jumps on short-term options was assessed by comparing the J model with the SV model and the SVJ model from three perspectives (a) in-sample pricing, (b) out-of-sample pricing, and (c) hedging effectiveness. It is concluded that the jump component has only a marginal effect, and the stochastic-volatility component is of the most importance even for pricing and hedging short-term options. The SVJ model performs the best for in-sample pricing, out-of-sample pricing, and hedging effectiveness. The J model shows the worst performance, and the differences between the SVJ and the SV models are small. Thus, even for short-term index options, jumps turn out to be insignificant.

APPENDIX

The characteristic functions $\hat{f}_j$ for the SVJ model are respectively given by

$$\begin{aligned}\hat{f}_1 = \exp\Bigg[&-i\phi\ln[B(t,\tau)] - \frac{\theta_\nu}{\sigma_\nu^2}\Bigg[2\ln\Bigg(1 - \frac{[\xi_\nu - \kappa_\nu + (1+i\phi)\rho\sigma_\nu](1-e^{-\xi_\nu\tau})}{2\xi_\nu}\Bigg)\Bigg] \\ &- \frac{\theta_\nu}{\sigma_\nu^2}[\xi_\nu - \kappa_\nu + (1+i\phi)\rho\sigma_\nu]\tau + i\phi\ln[S(t)] \\ &+ \lambda(1+\mu_J)\tau\lfloor(1+\mu_J)^{i\phi}e^{(i\phi/2)(1+i\phi)\sigma_J^2} - 1\rfloor - \lambda i\phi\mu_J\tau \\ &+ \frac{i\phi(i\phi+1)(1-e^{-\xi_\nu\tau})}{2\xi_\nu - [\xi_\nu - \kappa_\nu + (1+i\phi)\rho\sigma_\nu](1-e^{-\xi_\nu\tau})}V(t)\Bigg]\end{aligned} \tag{A-1}$$

and

$$\begin{aligned}\hat{f}_2 = \exp\Bigg[&-i\phi\ln[B(t,\tau)] - \frac{\theta_\nu}{\sigma_\nu^2}\Bigg[2\ln\Bigg(1 - \frac{[\xi_\nu^* - \kappa_\nu + i\phi\rho\sigma_\nu](1-e^{-\xi_\nu^*\tau})}{2\xi_\nu}\Bigg)\Bigg] \\ &- \frac{\theta_\nu}{\sigma_\nu^2}[\xi_\nu^* - \kappa_\nu + i\phi\rho\sigma_\nu]\tau + i\phi\ln[S(t)] \\ &+ \lambda(1+\mu_J)\tau\lfloor(1+\mu_J)^{i\phi}e^{(i\phi/2)(i\phi-1)\sigma_J^2} - 1\rfloor - \lambda i\phi\mu_J\tau \\ &+ \frac{i\phi(i\phi-1)(1-e^{-\xi_\nu^*\tau})}{2\xi_\nu^* - [\xi_\nu^* - \kappa_\nu + i\phi\rho\sigma_\nu](1-e^{-\xi_\nu^*\tau})}V(t)\Bigg]\end{aligned} \tag{A-2}$$

where

$$\xi_\nu = \sqrt{[\kappa_\nu - (1 + i\phi)\rho\sigma_\nu]^2 - i\phi(1 + i\phi)\sigma_\nu^2}$$
$$\xi_\nu^* = \sqrt{[\kappa_\nu - i\phi\rho\sigma_\nu]^2 - i\phi(i\phi - 1)\sigma_\nu^2}$$

The characteristic functions for the SV model can be obtained by setting $\lambda = 0$ in (A-1) and (A-2).

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